ESD-TDR-63-645

SPECIAL PERTURBATIONS WEIGHTED DIFFERENTIAL CORRECTION PROGRAM DOCUMENT

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-645

11 DECEMBER 1963

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496L SYSTEM PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. HANSCOM FIELD, BEDFORD MASS.

Prepared under Contract No. AF 19(628)-562 by Aeronutronic, a Division of Ford Motor Company, Newport Beach, California

FOR ERRATA

AD 426035

THE FOLLOWING PAGES ARE CHANGES

TO BASIC DOCUMENT

HEADQUARTERS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

LAURENCE G. HANSCOM FIELD BEDFORD, MASSACHUSETTS 01731

REPLY TO ATTN OF: ESTI/TSgt Wreck/4535 24 July 1964

SUBJECT: Page Revisions, ESD-TDR-63-645, SPWDC Program Document (AD 426 035)

TO: DDC
Cameron Station
Alexandria, Va.

AD-426035

1. ESD-TDR-63-645, subject, Special Perturbation Weighted Differential Correction Program, was transmitted to DDC on 31 December 1963.

2. The attached page revisions are forwarded for inclusion in the basic report.

FOR THE COMMANDER

Edward M. Doherty
EDWARD M. DOHERTY
Chief, Scientific and Technical
Information Division

1 Atch
Revisal pages,
70, 72, 72a, 75, 78, 79,
83a, 83b, & 83c

24 June 1964

SUMMARY OF CURRENT SPWDC INPUT CARD FORMATS*

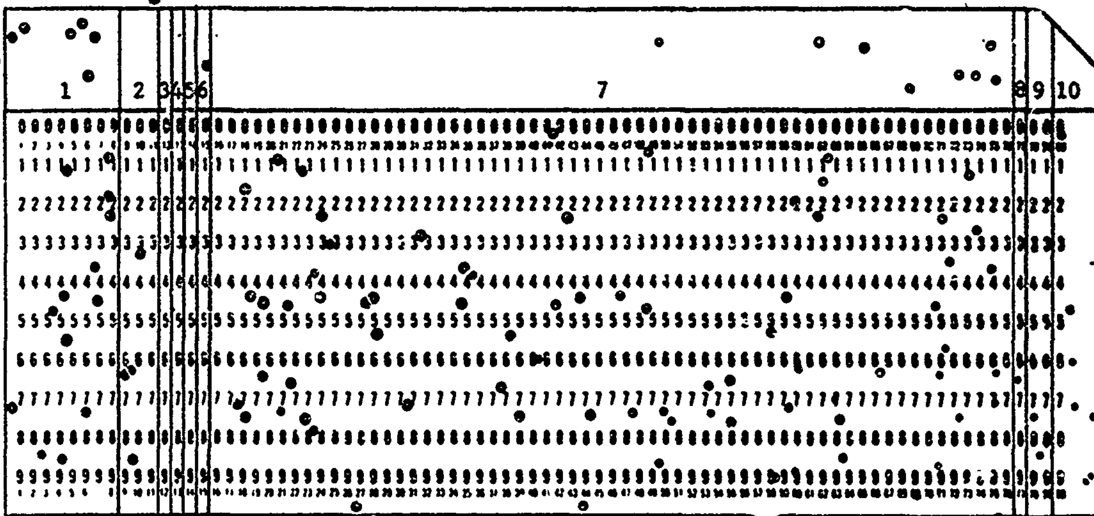
The programs SPWDC and ØBSWGTE have undergone changes by agencies both at Colorado Springs and at Bedford which invalidate some of the input card formats as described in the program document. Although these changes have been documented through program change notes, some of the program users have not become aware of the changes in program operation because of the limited distribution of change notes. In order to summarize the format changes in a single source and to provide for adequate distribution, this memo has been prepared. Figures showing the revised formats for the changed cards are included. It is suggested that these be inserted into each copy of the SPWDC Program Document, (ESD-TDR-63-645).

With respect to observation precession, it should be noted that if column 80 of an observation card is blank, and it is an optical type, (right-ascension, declination) then it is precessed in accordance with the standard system technique. That is to say, that if right-ascension and declination are observed, and the equipment type is not 16, then it is precessed from the year which is indicated in column 70 to the present, and if it is a field reduced Baker-Nunn observation (equipment type 16) it is precessed from 1855 to the present if the declination is greater than -22° and from 1875 if it is less.

* Prepared for the 496L SPO by Aeronutronic.

Aleb

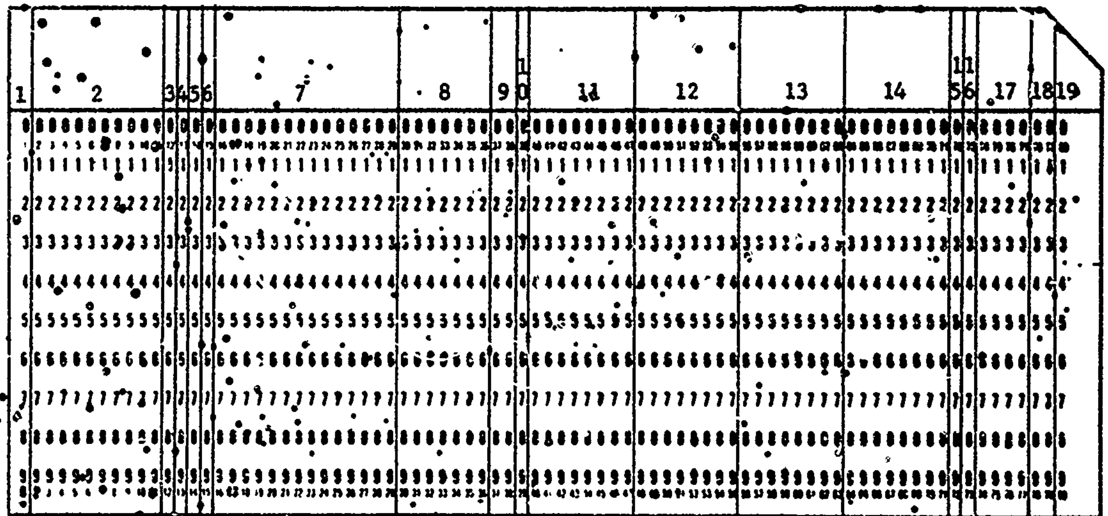
Reference: CN K-188 with attachments, CN T-335



Field	Column	Contents	Code
1	1 - 8	"SPWDCΔΔΔ"	
2	9 - 11	Satellite Number (optional)	
3	12	Auxiliary weight tape input (1=no, 0=yes)	
4	13	Raw data observation tape input (0=no, 1=yes)	
5	14	Must always be <u>blank</u> for SPWDC.	
6	15	Print input observations prior to DC. (0=no, 1=yes)	
7	16 - 76	Not used.	
8	77	Weighting flag. (1=no, 0=yes)	
9	78 - 79	Always "Δ0".	
10	80	Always "P".	

FIGURE 17. ØBSWGTE PARAMETER CARD

Reference: CN T-311

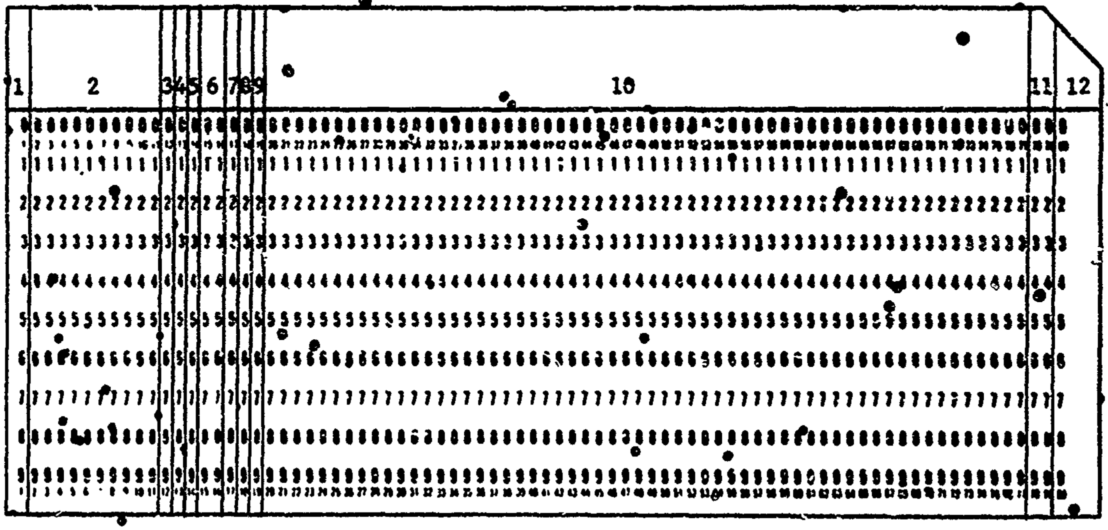


Field	Column	Contents	Code
1	1	Δt integration mode (Δ=variable, 1=fixed)	2
2	2 - 11	Δt (minutes) (positive makes correction epoch before obs., negative makes it after obs.)	3
3	12	Bulge perturbation flag	4
4	13	Drag perturbation flag	4
5	14	Radiation pressure perturbation flag	4
6	15	New epoch mode (0=rev, 1=time, 2=last ob.)	2
7	16 - 29	New epoch (day number in year and fraction, or revolution number.)	3
8	30 - 36	Elements to correct: n, a_{xN}, a_{yN}, U_o, Ω, i, m	4
9	37 - 38	Number of times to repeat correction.	2
10	39	Δq check flag (used only if 6 or 7 elements are being corrected.)	4
11	40 - 47	Maximum Δq (km) (used if col. 39 = 1.)	3
12	48 - 55	ABSMX (km)	3
13	56 - 63	ABSMX2 (km/sec)	3
14	64 - 71	n (rms multiplier for rejection.)	3

FIGURE 19. DIFFERENTIAL CORRECTION CONTROL CARD, Part 1 of 2

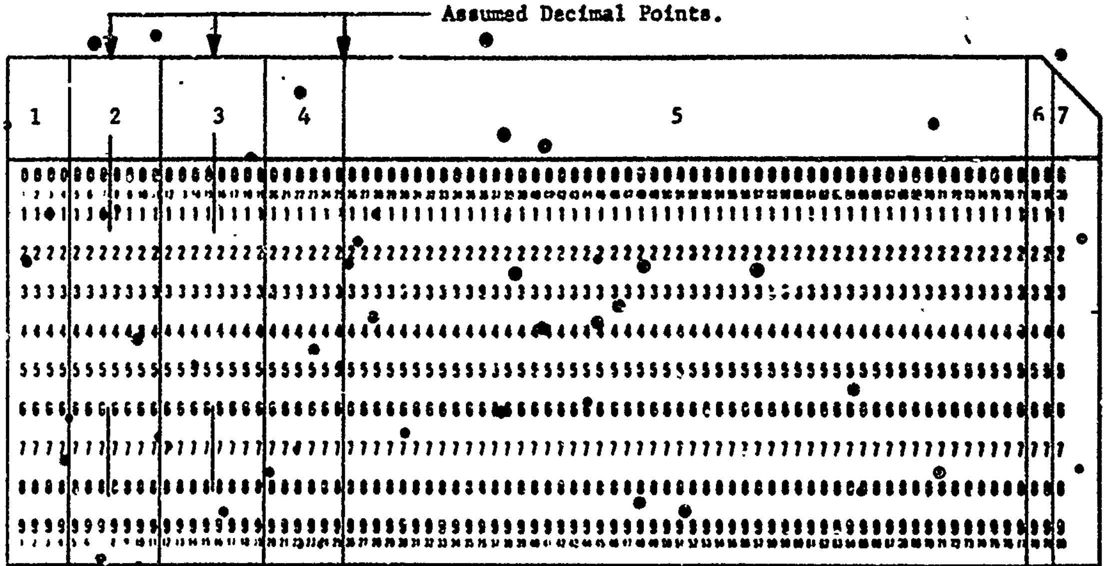
Field	Column	Contents	Code
15	72	Residual output flag (0=never, 1=first and last passes only, 2=all passes.)	2
16	73	Special "n and U" correction on first pass.	4
17	74 - 77	Relative change in successive rms's to determine convergence. (if blank, .05 is used.)	3
18	78 - 79	Always "Δ3".	
19	80	Always "P".	

FIGURE 19. DIFFERENTIAL CORRECTION CONTROL CARD, Part 2 of 2



Field	Column	Contents	Code
1	1	Δt integration mode (Δ=variable, 1=fixed)	?
2	2 - 11	Δt (minutes) (sign determined from PP card.)	3
3	12	Bulge perturbation flag	4
4	13	Drag perturbation flag	4
5	14	Radiation pressure perturbation flag	4
6	15 - 16	Print flag { 1 - t, $\underline{r}$, $\underline{\dot{r}}$ 2 - t, a, e, i, Ω, ω, U 4 - t, ϕ, λ_E, h } Add flags for combination of data.	2
7	17	Binary ephemeris tape output flag	4
8	18	Prediction reliability output flag	4
9	19	Punched cards (t, $\underline{r}$) (Also leaves data in core for later use by GIPAR)	4
10	20 - 77	Not used.	
11	78 - 79	Always "A6".	
12	80	Always "P".	

FIGURE 22. TIME PREDICTION CARD #1

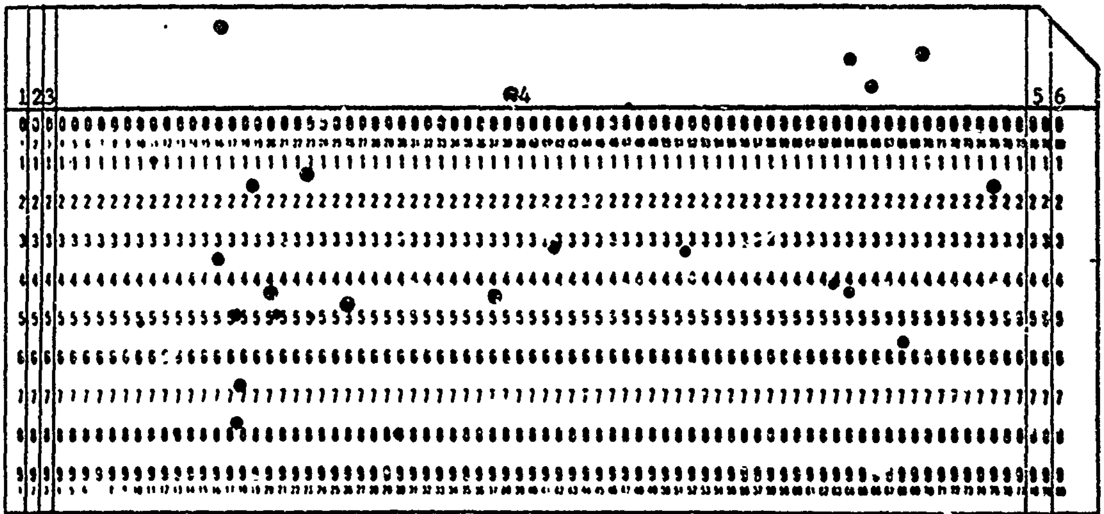


Field	Column	Contents	Code
1	1 - 4	Sensor number	1
2	5 - 11	Station latitude (deg. north) (assumed dec. pnt.)	3
3	12 - 19	Station longitude (deg. west) (assumed dec. pnt.)	3
4	20 - 25	Station altitude (meters) (assumed dec. pnt.)	3
5	26 - 77	Not used.	
6	78 - 79	Always "Δ9".	
7	80	Always "P".	

Note: Trailing zeros must be punched in fields 2 and 3.

FIGURE 25. STATION PASS PREDICTION CARD #2

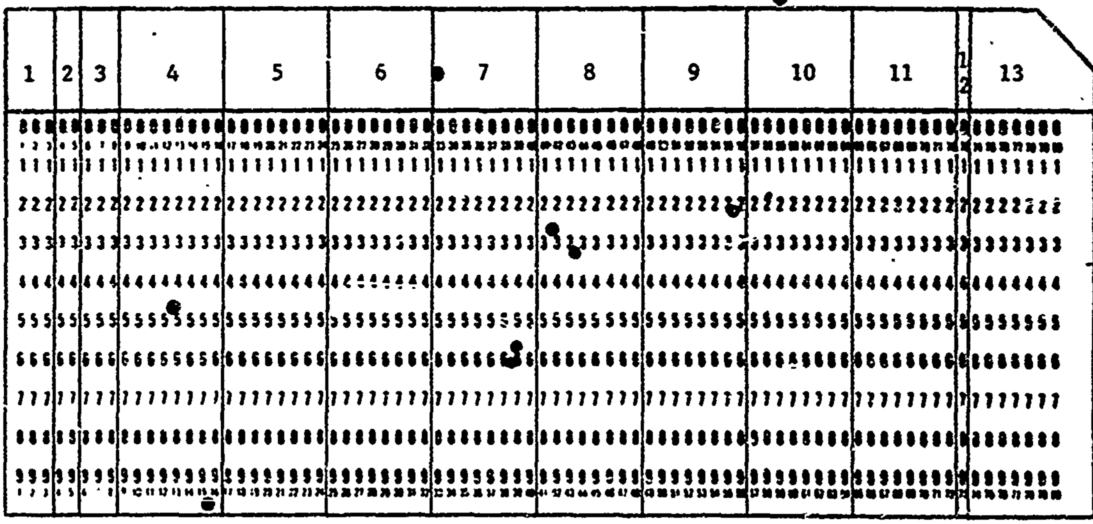
Reference: CN's T-227, T-248, T-276, T-286, T-299, & T-313.



Field	Column	Contents	Code
1	1	DC Flag	4
2	2	Prediction (0-no)(1-Time)(2-Station)	2
3	3	0 or blank - Print only Δt, U, and β for "n and U" correction pass. 1 - Print also $\Delta\rho$, $\rho\cos\delta\Delta\alpha$, $\rho\Delta\delta$, $\Delta\dot{\rho}$ for "n and U" correction pass. 2 - Print $\Delta\rho$, $\cos\delta\Delta\alpha$, $\Delta\delta$, $\Delta\dot{\rho}$, in degrees for all passes. 3 - Make only one pass. Additional residuals in U,V,W coordinates are computed and printed, as well as residual means and standard deviations for each of the sensors. 4 - Like option 3 except that residuals in observed quantities are punched on cards as well as printed. (Format given in CN T-313.)	2
4	4 - 77	Not used.	
5	78 - 79	Always "10".	
6	80	Always "P".	

FIGURE 26. PROGRAM EXECUTION CARD

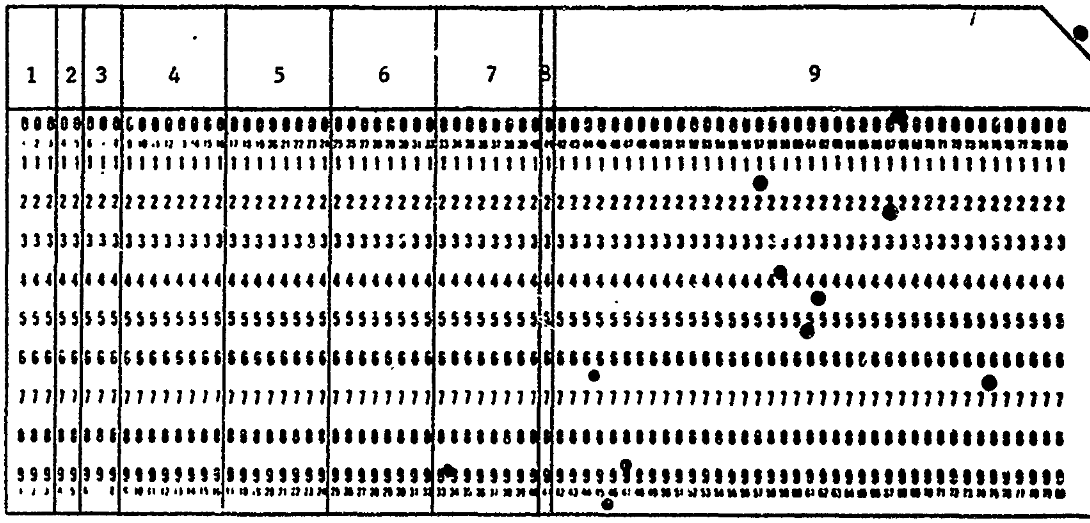
Reference: CN K-188 with attachments.



Field	Column	Contents	Code
1	1 - 3	Sensor number	1
2	4 - 5	Always "11",	
3	6 - 8	Not used.	
4	9 - 16	Always "CSGBIASA"	
5	17 - 24	Range standard deviation (km)	3
6	25 - 32	Range bias (km)	3
7	33 - 40	Range-rate standard deviation (km/sec)	3
8	41 - 48	Range-rate bias (km/sec)	3
9	49 - 56	Azimuth (or rt. asc.) standard deviation (deg)	3
10	57 - 64	Azimuth (or rt. asc.) bias (deg)	3
11	65 - 72	Elevation (or decl.) standard deviation (deg)	3
12	73	Multiple punch (11-8-2)	
13	74 - 80	Not used.	

FIGURE 30A. CSGBIAS FUNCTION WEIGHTING CARD #1

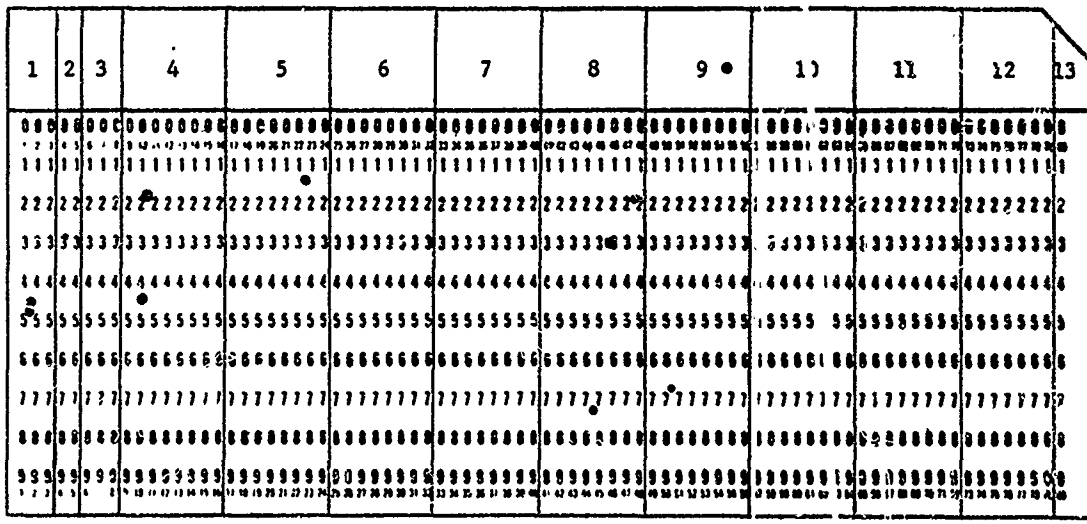
Reference: CN K-188 with attachments.



Field	Column	Contents	Code
1	1 - 3	Sensor number	1
2	4 - 5	Always "10".	
3	6 - 8	Not used.	
4	9 - 16	Always "CSGBIASA"	
5	17 - 24	Elevation (or decl.) bias (deg)	3
6	25 - 32	Observation time bias (sec)	3
7	33 - 40	Not used	
8	41	Multiple punch (11-8-2)	
9	42 - 80	Not used	

FIGURE 30B. CSGBIAS FUNCTION WEIGHTING CARD #2

Reference: CN K-112 with attachment.



Field	Column	Contents	Code
1	1 - 3	Sensor number	1
2	4 - 5	Always "10".	
3	6 - 8	Not used.	
4	9 - 16	Always "SIGTAB1Δ"	
5	17 - 24	Alternate value selector (anywhere in field) 0 or Δ - Primary value 1 - 1st alternate value if compiled in OBSWGTE 2 - 2nd alternate value if compiled in OBSWGTE	2
6	25 - 32	Scale factor. All tabular values of standard deviations are multiplied by this factor. (If blank, a value of one is assumed.)	3
7	33 - 40	Range bias (km)	3
8	41 - 48	Elevation (or decl.) bias. (deg)	3
9	49 - 56	Azimuth (or rt. asc.) bias. (deg)	3
10	57 - 64	Range-rate bias. (km/sec)	3
11	65 - 72	Observation time bias. (min.)	3
12	73 - 79	Not used.	
13	80	Multiple punch (11-8-2)	

FIGURE 30C. SIGTAB1 FUNCTION WEIGHTING CARD

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FOREWORD

Contractor's Report Publication No. U-2370

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SPECIAL PERTURBATIONS WEIGHTED DIFFERENTIAL CORRECTION PROGRAM DOCUMENT

ABSTRACT

This report describes a computer program which differentially corrects and/or predicts the orbit of a geocentric satellite. The dynamics of the program are based on a variation of parameters formulation with the perturbative acceleration being numerically integrated. A weighted correction process is incorporated which considers the relative accuracy of the observational data in a least-squares fit. Included in the document are the program description, complete formulation, program operating instructions, flow diagrams, and test cases.

CONTENTS

ILLUSTRATIONS

SECTION 1

INTRODUCTION

Aeronutronic has developed an operational weighted differential correction program, using the special perturbations variation of parameters approach. Operating under the 496L B-2 Semi-automatic Programming System, the program will accept sensor observations, assign weights to the data, perform a differential correction and predict future position and velocity.

The purpose of the Special Perturbations Weighted Differential Correction (SPWDC) Program is to provide highly precise predictions of satellite position. The operational capability to utilize weights when performing a differential correction in addition to the highly accurate Special Perturbations formulation will provide far more accurate position prediction than the Simplified General Perturbations approach. The latter concept was designed to provide sufficient accuracy to maintain surveillance on all earth satellites with minimum computer time requirements. This surveillance mission requires only such accuracy as to ensure acquisition and recognition of every satellite by sensors capable of observing it.

The SPWDC Program, therefore, supplies a means for correcting orbital parameters and predicting in an extremely accurate fashion for missions requiring precise position determination. Operationally, it functions in the B-2 System running in the Schedule Tape Mode.

Section 2 of this document is a description of the SPWDC Program. The formulation by which the program generates the numerically integrated position is given in Section 3 of the report. Operating instructions are detailed in Section 4, with parameter card formats shown in Appendix I. The main flow of the program is diagramed in Section 5, and detailed flow data in Appendix II. Compiled test cases are outlined in Section 6. A glossary of terms will be found in Appendix III.

SECTION 2

PROGRAM DESCRIPTION

The SPWDC Program contains three main functions. These functions, shown schematically in Figure 1, are: OBSERVATION WEIGHTING, DIFFERENTIAL CORRECTION, and PREDICTION. Each of these functions is described here, in terms of its general features, required parameters, and function controls both internal and output. Required parameters are those numerical quantities describing the physical conditions to be simulated in the program. Internal controls are those quantities which specify the program logic to be executed, and output controls are those quantities prescribing the choice of available output.

2.1 OBSERVATION WEIGHTING

The Observation Weighting function of the program is to reformat input observations, assigning to each observed quantity its standard deviation. These accuracy estimates are provided to the routine in three ways:

(1) By assigning a constant accuracy estimate for all observations from a given sensor;

(2) By assigning an accuracy estimate to each observation being used; or

(3) By indicating a programmed weight function which is used to compute the observation parameters.

The assignment of weight factors to the observations may be made by any combination of the above methods, with the restriction that any given sensor will use only one of three choices.

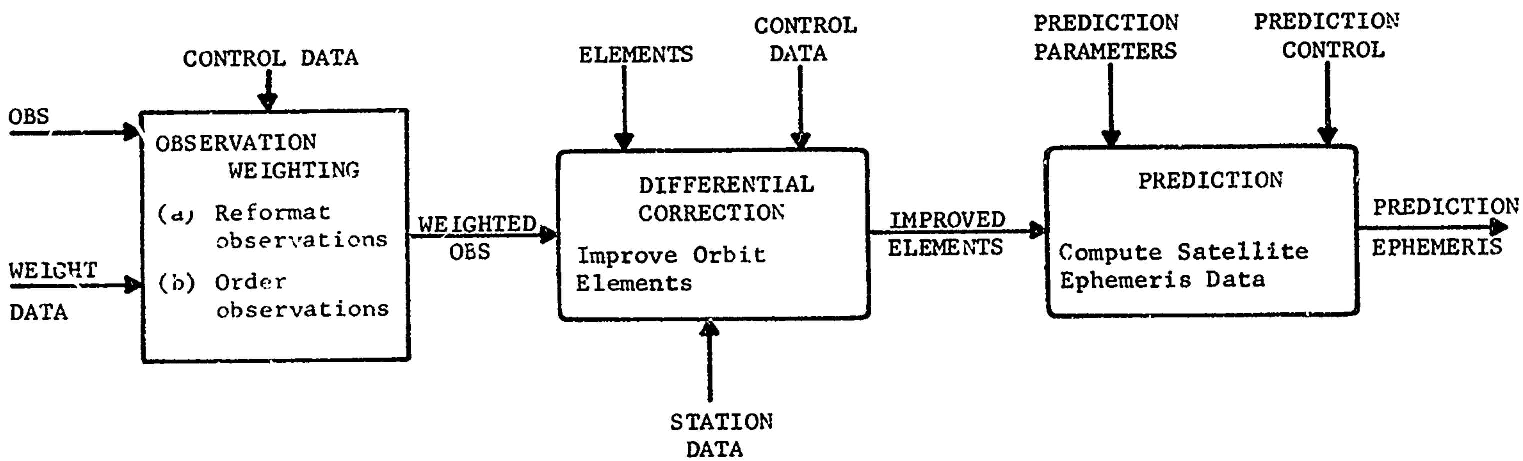


FIGURE 1. SPWDC PROGRAM FUNCTIONS

After the complete set of observations has been examined and each weight assignment has been made, the observations are ordered chronologically. This feature is necessary in the Observation Weighting function in order to ensure an efficient correction process. Since the correction program must represent the observations at each actual observation time by numerically integrating to that time, it is highly desirable that the observations be ordered in time so the integration takes place entirely in one direction.

a. Observation Weighting Parameters

Observations are supplied by:

(1) Standard SPS Observation Card, or

(2) SRADU tape.

Weighting information is supplied by the Weighting Tape (see Section 4).

b. Observation Weighting Controls

Internal controls are as follows:

(1) Weighting information may be picked up either from the Weighting Tape, or a set of accuracy estimates may be compiled into the Observation Weighting Program in the weight by sensor mode. Normally, the former selection will be made; however, it may be to the user's advantage to program the sensor weight values for automatic internal assignment.

(2) Weighting data may or may not be assigned to the observations. It should be noted that if weights are to be assigned, they must be assigned to <u>all</u> observations. It is not possible to mix weighted and unweighted data.

(3) A selection of weighting mode is required to indicate the choice of weighting by sensor, by observation, or by function.

No output controls are available to the Observation Weighting function.

2.2 DIFFERENTIAL CORRECTION

The Differential Correction function accepts the reformatted observations from the Observation Weighting function and performs a weighted or unweighted differential correction on the orbit element set. Ephemeris computation is carried out in a special perturbations variation of parameters formulation. This formulation numerically integrates the perturbative accelerations influencing the parameters of an instantaneous two-body reference orbit.

The numerical integration is performed by a fourth-order Runge-Kutta integration routine. A fixed step size may be specified for this numerical integration, or it may be used in a variable mode. In the variable mode, the initial step size is specified and subsequently increased or decreased internally, based on error control parameters.

Perturbations handled by the program include: zonal harmonic bulge, atmospheric drag, and solar radiation pressure. These effects may be controlled to incorporate any combination in a particular case.

A correction can be obtained for any of the six orbit elements, and may also include the satellite mass. This latter parameter actually represents a correction on the ballistic parameter $\frac{C_D A}{m}$, where C_D is the drag coefficient, A is the satellite cross-sectional area, and m is the satellite mass.

First-order partials relating variations in observed quantities to variations in orbit parameters are the basis of the correction equations. The correction for mass, or ballistic parameter, uses an equation derived on the assumption of near circular eccentricity. Therefore, for the general case, a variant orbit approach is used, whereby the satellite mass is varied and a complete integration is performed. This ephemeris is then used to form numerical partials between the observed quantities and the variation in mass.

Special correction passes may be taken. For instance, it may be desirable to correct the mean motion initially before attempting a complete correction. This will reduce the timing residual and provide a better fit on subsequent correction passes. Also, an incernal check has been built in which examines the change in the perigee distance between successive corrections. If the perigee distance varies too

greatly, the correction is automatically repeated without correcting the orbital eccentricity. This prevents a possible divergent correction which may occur due to non-linear drag effects.

The correction process will repeat up to a maximum number of iterations, or until the root-mean-square (RMS) of the accepted residuals has converged within 5% comparing successive values. Accepted residuals are those that are not rejected on the basis of an absolute maximum or a relative check based on a limit of 1.5 times the residual RMS. In a weighted differential correction this is the weighted RMS; however, the convergence check is still against the unweighted value. In addition, weighted runs always perform an unweighted correction on the first iteration in order to fit the elements as well as possible before applying weight data. An option is also provided, enabling the post correction epoch to be updated to a specific revolution number or time.

a. Differential Correction Parameters

Orbit elements are supplied by:

(1) The standard card format, or

(2) The SEAI file.

Vehicle data are mass, diameter, and reflectivity.

b. Differential Correction Controls

Internal controls are as follows:

(1) The integration mode, variable or fixed, is specified.

(2) The integration step size is specified. If the mode is variable, then this denotes the initial step size.

(3) Selection of perturbations to be used: bulge, drag, radiation pressure.

(4) Specification of the new epoch:

(a) Revolution number (actual number, not referenced to current epoch)

(b) Time (absolute), or

(c) Time of last observation.

(5) Specification of the elements to be corrected Any combination may be selected; however, in some instances the corrections are not independent and will influence the other parameters.

(6) Special correction passes:

(a) Correct mean motion only on first pass

(b) Maximum value allowed for change in perigee distance

(7) Specification of rejection criteria:

(a) Absolute maximum for displacement residuals,

(b) Absolute maximum for range-rate residuals, and

(c) Multiplying factor applied to the residual RMS for relative rejection.

(8) Error co ols for Runge-Kutta integration:

(a) Absolute controls

(b) Relative controls

One output control is available to the Differential Correction function. This establishes the frequency of output of the residuals as: never, first and last set, or every set.

2.3 PREDICTION

The Prediction function of the program may be used to obtain future position and velocity data from an input element set or a corrected set from the Differential Correction function. Two methods are provided for the specification of the prediction time. A series of prediction points may be specified by time or by the closest point of approach to some location on the earth.

Ephemeris computation in the Prediction function is carried out in the same manner as in the Differential Correction, i.e., using a special perturbations variation of parameters formulation where the integration is according to a fourth-order Runge-Kutta process. The mode of integration, integration step size, and the perturbations to be considered must be supplied.

In addition to the prediction, this function has the capability of estimating the reliability of the prediction point. The reliability check computes the variance in both inertial and radial, transverse and orthogonal components of position and velocity. It is based on the variance-covariance matrix of orbit elements derived from the Differential Correction function.

a. Prediction Parameters

Prediction by time requires the following parameters:

(1) Initial prediction time (absolute)

(2) Prediction interval (minutes)

(3) Number of points to be obtained

Prediction by station pass requires the following parameters:

(1) A station number identification. This would pertain to the SEAI File designation if the location is to be one of the SPADATS sensors. Otherwise, any number designation will suffice.

(2) Latitude, longitude (west), and altitude of the station, required if the SEAI File is not being used.

(3) The length of time to simulate the ephemeris on either side of the closest point of approach.

(4) The maximum elevation angle below which no ephemeris data will be obtained. This takes precedence over item (3) above.

(5) The interval of time to be used in obtaining the ephemeris.

(6) The number of passes to be considered.

b. <u>Prediction Controls</u>

Internal controls are as follows:

(1) The integration mode, variable or fixed, is specified.

(2) The integration step size is specified. As in the Differential Correction, if the mode is variable, then this denotes the initial step size.

(3) Selection of perturbations to be used: bulge, drag, radiation pressure.

Output controls are as follows:

(1) Print any combination of

(a) time, position, and velocity (t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$)

(b) time, osculating elements (t, a, e, i, Ω, ω, L)

(c) time, ground track data (t, satellite latitude, satellite east longitude, and satellite altitude)

(d) time, look angles (t, range, range-rate, azimuth, elevation) available only for station pass prediction

(2) Print reliability estimate consisting of the standard deviation in the position and velocity predictions in inertial coordinates and orbital coordinates.

(3) Prepare a binary ephemeris tape containing t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$.

(4) Punch position cards containing t, x, y, z.

SECTION 3

FORMULATION

The Special Perturbation Weighted Differential Correction (SPWDC) Program employs a variation of parameters formulation to generate numerically integrated positions. This ephemeris is computed using the $\underline{N}$, $\underline{M}$ parameters, namely, mean motion, n_o, mean longitude, L_o; two parameters embodying eccentricity, e, and argument of perigee, ω : $a_{xNo} = e \cos \omega$ and $a_{yNo} = e \sin \omega$; and a vector defining the angular momentum, $\underline{h}_o = \sqrt{p_o}\, \underline{W}$, whose components are

$$h_{x_o} = \sqrt{p_o} \quad \sin \Omega_o \sin i_o$$

$$h_{y_o} = \sqrt{p_o} \quad \cos \Omega_o \sin i_o$$

$$h_{z_o} = \sqrt{p_o} \quad \cos i_o$$

where p_o is the semi-latus rectum, Ω_o is the right ascension of the ascending node, and i_o is the orbital inclination (see Figure 2). The subscript identifies these quantities as the orbital parameters at epoch, t_o. These parameters do not exhibit singularities at zero eccentricity. A parameter adoption in the differential correction section of the program effectively limits the program application to orbits having inclinations greater than two degrees.

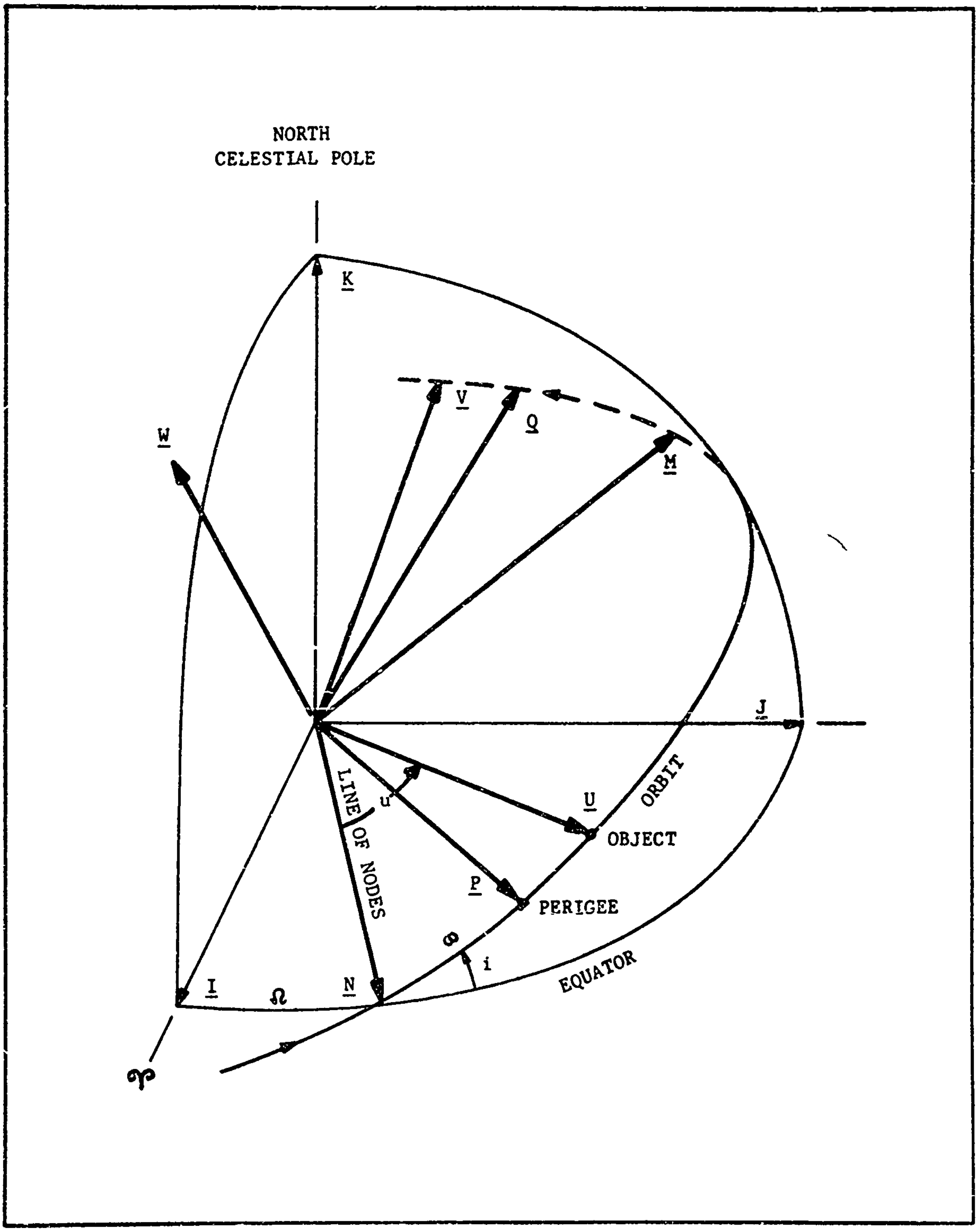


FIGURE 2. PROJECTION OF ORBIT ON CELESTIAL SPHERE, WITH ORIENTATION UNIT VECTORS AND ANGLES DISPLAYED

The formulation accounts for perturbations due to drag forces and the primary effects of the Earth's asphericity[1]* and to radiation pressure[2]. The elements L_o, n_o, a_{xNo}, and $\underline{h}_o$ vary from point to point on the resulting ephemeris in such a manner that they satisfy a system of differential equations in the dependent variables, the elements, and the independent variable, time, t. The solution to the system of differential equations is achieved in a numerical integration process, executed by means of the Runge-Kutta procedure. The program proceeds to evaluate the elements from point to point by using either a fixed grid size for the variable t or a variable grid size controlled by the program.

The program will function in a differential correction mode and/or an ephemeris mode. In the differential correction mode, a given set of elements is improved and updated by means of a least-squares fit to a set of observations. These observations may be a weighted set or an unweighted set with the correction functioning accordingly. In the prediction mode, future position and velocity data are provided by integrating to the specified point, using the corrected set of elements or an element set introduced for the purpose of prediction only.

3.1 OBSERVATION WEIGHTING

One of the basic considerations to be included in the development of accurate Special Perturbations programs is that of assignment of weighting factors to all observations for subsequent use in the differential correction process. Because of the restrictive core allocation imposed by the operational system, the weight assignment is performed independent of the remainder of the processing.

The two main design features of the resulting proprocessing program are (1) to reformat the observational data, adding weights for all the quantities, and (2) to order the observations in time, for either a forward or backward integration.

These design criteria were incorporated into the observation weighting function (OBSWGT) of SPWDC. The functional flow between OBSWGT and SPWDC is shown in Figure 3. The general specifications of the program are:

*Superscripts indicate References at end of document.

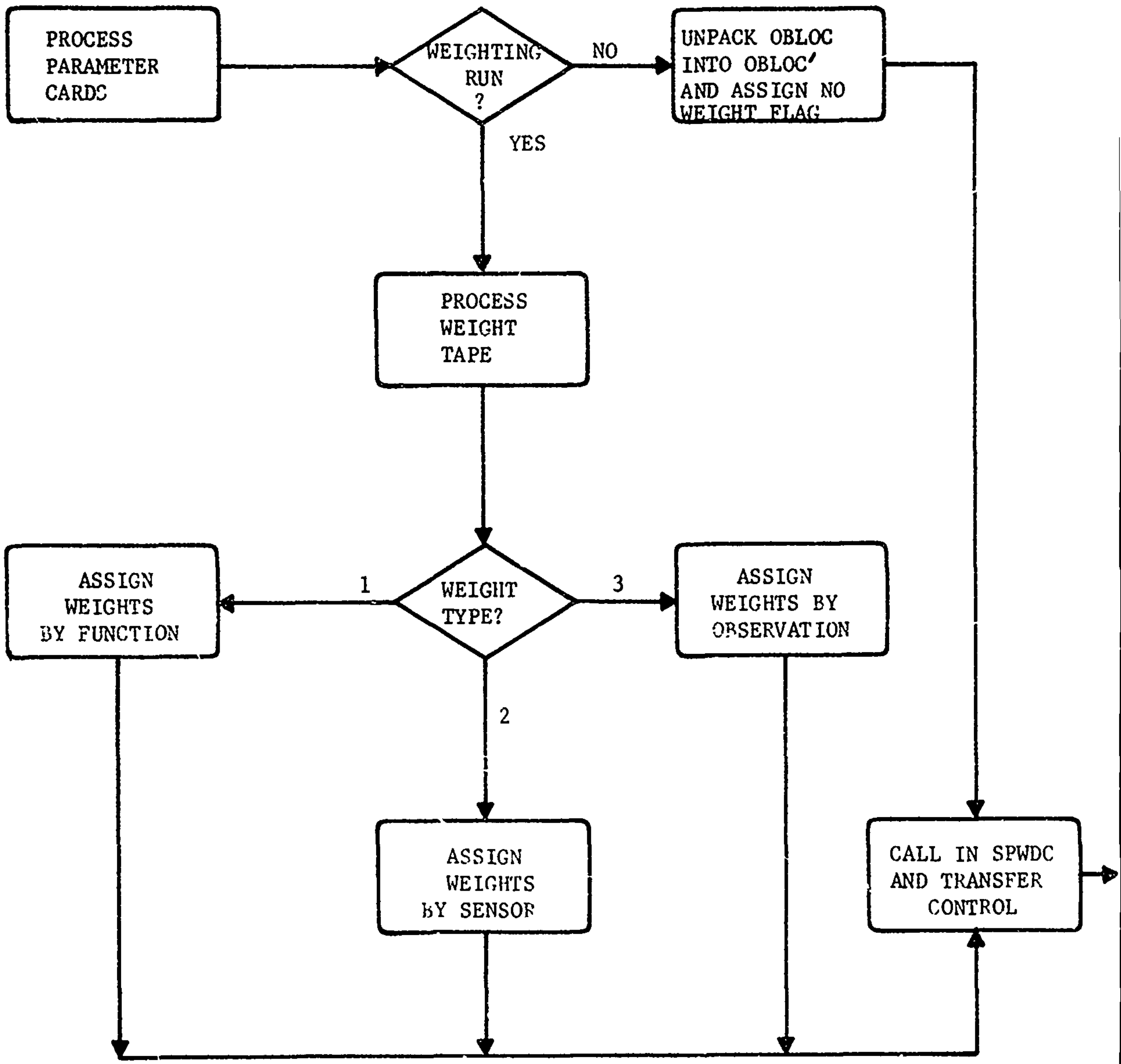


FIGURE 3. OBSWGT FUNCTIONAL FLOW

(1) OBSWGT will operate before the Special Perturbation Program. OBSWGT will apply weights to all the observations, leave the observations in core, and call (and exit to) SPWDC from the master program tape.

(2) OBSWGT will operate as a schedule tape job. Standard input will consist of combinations of parameter, element, sensor and observations cards.

(3) Additional input data will be contained on an auxiliary tape. Data on this tape will contain weight factors for each sensor and/or observation or will contain controls for determining the method to be used for computing the weights. Operationally, this will be known as the Weighting Tape.

   (a) A single set of weights (at most, four quantities) can be specified for a sensor. These weights then apply to all observations reported by the sensor.

   (b) A single set of weights can be specified for each observation.

   (c) A routine can be specified by sensor to compute weight factors for each observation reported by the sensor.

(4) A subroutine will be made available to the Special Perturbation program to retrieve the observation quantities, weight factors, and to retrieve the required sensor data.

(5) The routines for computing weight factors (mentioned in (3) (c) above) will have a standard calling format and be contained as a subroutine in the OBSWGT program. These routines can only be added, deleted, or changed by recompilations. However, the parameters used by the routines can be changed (specified as input quantities) prior to each run.

(6) All weighting procedures will be general so that the specific function to be applied is determined on the Weighting Tape by pairing a sensor with a function as an input quantity. In this way routines can apply to more than one sensor, reducing duplication of routines.

(7) A special bypass mode will be available to process observations without applying weight factors.

3.2 VARIATION OF PARAMETERS EPHEMERIS COMPUTATION

The following paragraphs detail the formulation of the representation, accelerations, and integration of the orbit. The representation calculates position and velocity at a given time from the orbital parameters. The accelerations are computed from the force field formulation. The integration is performed upon the derivatives of the orbital parameters.

a. Representation

(1) Initialization

Prior to any calculations, the epoch elements must be determined from the input. Since the input is either in the form of 7 element cards or from the SEAI tape, the system subroutine NXTELM is used.

Given the epoch parameter set, the following computations are performed:

$$p_o = \underset{\sim}{h}_o \cdot \underset{\sim}{h}_o$$

$$\underset{\sim}{W}_o = \frac{\underset{\sim}{h}_o}{\sqrt{p_o}}$$

$$\cos i_o = W_{z_o}$$

$$\sin i_o = \sqrt{1 - W_{z_o}^{\;2}}$$

$$i_o = \arctan \left[\frac{\sin i_o}{\cos i_o}\right]$$

$$\sin \Omega_o = \frac{W_{x_o}}{\sin i_o}$$

$$\cos \Omega_o = \frac{-W_{y_o}}{\sin i_o}$$

$$\Omega_o = \arctan \frac{\sin \Omega_o}{\cos \Omega_o}$$

$$e_o^2 = a_{xN_o}^{\ 2} + a_{yN_o}^{\ 2}$$

$$a_o = \frac{p_o}{1 - e_o^2}$$

$$U_o = L_o - \Omega_o \qquad \text{if } W_{z_o} \geq 0$$

$$\phantom{U_o = {}} L_o + \Omega_o \qquad \text{if } W_{z_o} < 0$$

$$n_o = \frac{\sqrt{\mu}\, k_e}{a_o^{3/2}}$$

$$a_{x_o} = a_{xN_o} \cos\Omega_o - a_{yN_o} \sin\Omega_o \cos i_o$$

$$a_{y_o} = a_{xN_o} \sin\Omega_o + a_{yN_o} \cos\Omega_o \cos i_o$$

$$a_{z_o} = a_{yN_o} \sin i_o$$

$$q_o = a_o (1 - e_o)$$

If the input mass parameter = 0, mass is set equal to 1. The quantity 1/mass is formed.

(2) Position and Velocity

At each time point, whether at an integration step, observation time or prediction point, a similar initialization takes place. Then, the program goes on to compute the geocentric position ($\underline{r}$) and velocity ($\dot{\underline{r}}$) of the satellite.[1]

Given $\underline{a}$, $\underline{h}$, $\underline{L}$, compute $\underline{r}$, $\dot{\underline{r}}$

$$p = \underline{h} \cdot \underline{h}$$

$$e^2 = \underline{a} \cdot \underline{a}$$

$$a = \frac{p}{1 - e^2}$$

$$n = \frac{k_e \sqrt{\mu}}{a^{3/2}}$$

$$\underline{W} = \frac{\underline{h}}{\sqrt{p}}$$

$$W_z = \cos i$$

$$M_z = \sqrt{1 - W_z^{\,2}} = \sin i$$

$$N_x = -\frac{W_y}{M_z}$$

$$M_y = N_x W_z$$

$$N_y = \frac{W_x}{M_z}$$

$$M_x = -N_y W_z$$

$$N_z = 0$$

$$a_{xN} = \underline{a} \cdot \underline{N}$$

$$a_{yN} = \underline{a} \cdot \underline{M}$$

$$\Omega = \arctan \frac{N_y}{N_x} = \arctan \frac{W_x}{-W_y}$$

$$U = L - \Omega \quad \text{if} \quad W_z \geq 0$$

$$\phantom{U = {}} L + \Omega \quad \text{if} \quad W_z < 0$$

$$0 \leq U < 2\pi$$

Kepler's equation is solved by iteration using the Newton-Raphson method with an initial guess for $(E+\omega)$ of U [i.e., $(E+\omega)_1 = U$]

$$(E+\omega)_{i+1} = (E+\omega)_i - \frac{[U + e \sin E_i - (E+\omega)_i]}{e \cos E_i - 1} \quad \text{radians}$$

where

$$e \cos E_i = a_{xN} \cos (E+\omega)_i + a_{yN} \sin (E+\omega)_i$$

$$e \sin E_i = a_{xN} \sin (E+\omega)_i - a_{yN} \cos (E+\omega)_i$$

The iteration is concluded when

$$\left| (E+\omega)_{i+1} - (E+\omega)_i \right| < 10^{-8}$$

If, after 50 iterations, the criterion is not met, the run is terminated. A comment to this effect is written on the output tape.

After Kepler's equation is solved, the calculations continue:

$$r = a\,(1 - e \cos E)$$

$$\dot{r} = \frac{\sqrt{\mu . a}}{r} (e \sin E)$$

$$r\dot{v} = \frac{\sqrt{\mu\, a}}{r} \sqrt{1 - e^2}$$

$$\cos u = \frac{a}{r} \left[\cos (E + \omega) - a_{xN} + a_{yN} \frac{e \sin E}{(1 + \sqrt{1 - e^2})}\right]$$

$$\sin u = \frac{a}{r} \left[\sin (E + \omega) - a_{yN} - a_{xN} \frac{e \sin E}{(1 + \sqrt{1 - e^2})}\right]$$

$$\underline{U} = \underline{N} \cos u + \underline{M} \sin u$$

$$\underline{V} = - \underline{N} \sin u + \underline{M} \cos u$$

$$\underline{r} = r\, \underline{U}$$

$$\underline{\dot{r}} = \dot{r}\, \underline{U} + r\dot{v}\, \underline{V}$$

b. Drag

To compute the drag perturbation, it is first necessary to calculate H, the altitude above an oblate spheroid:

$$H = (r - 1) - \frac{3}{2} f^2 (U_z)^4 + (f + \frac{3}{2} f^2) U_z{}^2$$

For this altitude the 1962 U.S. Standard Atmosphere Table is searched for the density (ρ_r) and molecular weight (M_e).

Values above 700 km are obtained by linear extrapolation.

The acceleration is obtained by the following computation(1):

$$\nu_x = \dot{x} + y\dot{\theta} \quad \text{where } \dot{\theta} = 0.058\,834\,47 \text{ radians/k}_e \text{ min}$$

$$\nu_y = \dot{y} - x\dot{\theta}$$

$$\nu_z = \dot{z}$$

$$\nu = \sqrt{\nu_x{}^2 + \nu_y{}^2 + \nu_z{}^2}$$

$$T_s = \left[\frac{\rho_r C_D (V_{co})^3 \nu^3}{4 e\sigma_s} + (300)^4\right]^{1/4}$$

$$C = \frac{6.972 \times 10^9 \ \nu d}{C_{D_o} \sqrt{M_e T_s}}$$

$$\text{if } C_\rho > 1418, \quad C_D = C_{D_o}$$

otherwise,

$$C_D = C_{D_o} (1 + 1.1739130 \ e^{-C\sigma})$$

$$D_c = C_D \rho_r \nu \left(- \frac{K \pi d^2}{8m}\right)$$

$$\ddot{x}_D = D_c \, \nu_x$$

$$\ddot{y}_D = D_c \, \nu_y$$

$$\ddot{z}_D = D_c \, \nu_z$$

c. Asphericity (Bulge)

The acceleration ($\underline{\ddot{r}}_B$) due to the zonal harmonics of the Earth's gravitational potential are calculated by

$$\ddot{x}_B = x \left[\frac{\mu}{r^3} \sum_{n=2}^{7} J_n \left(\frac{a_e}{r}\right)^n P'_{n+1} (U_z) \right]$$

$$\ddot{y}_B = y \left[\frac{\mu}{r^3} \sum_{n=2}^{7} J_n \left(\frac{a_e}{r}\right)^n P'_{n+1} (U_z) \right]$$

$$\ddot{z}_B = z \left[\frac{\mu}{r^3} \sum_{n=2}^{7} J_n \left(\frac{a_e}{r}\right)^n P'_{n+1} (U_z) \right]$$

$$- \frac{\mu}{r^2} \sum_{n=2}^{7} J_n \left(\frac{a_e}{r}\right)^n P'_n (U_z)$$

where $\quad P_o (U_z) = 1, \quad P_1 (U_z) = U_z$

and

$$\left.\begin{aligned} P_n (U_z) &= \frac{1}{n} \left[(2n - 1) U_z P_{n-1} (U_z) - (n-1) P_{n-2} (U_z) \right] \\ P'_n (U_z) &= \frac{1}{U_z^2 - 1} \, n \left[U_z P_n (U_z) - P_{n-1} (U_z) \right] \end{aligned}\right\} \quad n \geq 2$$

The coupling factors used are[5]:

$$\left.\begin{aligned} J_2 &= 1082.50 \\ J_3 &= -2.553 \\ J_4 &= -1.807 \end{aligned}\right\} \times 10^{-6} \qquad \left.\begin{aligned} J_5 &= -0.083 \\ J_6 &= +0.416 \\ J_7 &= -0.415 \end{aligned}\right\} \times 10^{-6}$$

d. Radiation Pressure

The acceleration due to direct solar radiation pressure, $\underline{\ddot{r}}_{RP}$, is formulated as detailed in Reference 2. Initially, the mean longitude of the sun at the beginning of the current year $L_{\odot o}$ is found by the system subroutine TLC[3]:

$$L_{\odot} = L_{\odot o} + n_{\odot} (t - t_o)$$

$$M_{\odot} = L_{\odot o} + n_{\odot} (t - t_o) - \pi_{\odot}$$

$$\ell_{\odot} = L_{\odot o} + n_{\odot} (t - t_o) + 2 e_{\odot} \sin M_{\odot} + \frac{5}{4} e_{\odot}^2 \sin 2M_{\odot}$$

$$L_{x_{\odot}} = \cos \ell_{\odot}$$

$$L_{y_{\odot}} = \cos \epsilon \sin \ell_{\odot}$$

$$L_{z_{\odot}} = \sin \epsilon \sin \ell_{\odot}$$

$$\cos \psi = \frac{\underline{L}_{\odot} \cdot \underline{r}}{r}$$

$$\ddot{x}_{RP} = \left(\frac{F_{\odot}}{M}\right) L_{x\odot}$$

$$\ddot{y}_{RP} = \left(\frac{F_{\odot}}{M}\right) L_{y\odot}$$

$$\ddot{z}_{RP} = \left(\frac{F_{\odot}}{M}\right) L_{z\odot}$$

If $\cos \psi > 0$, the satellite is illuminated.

If $\cos \psi < 0$, form $\sin (\psi + \eta) = \sin \psi \cos \eta + \sin \eta \cos \psi$

$$= \sqrt{(\cos^2 \psi - 1)\left(\frac{1}{r^2} - 1\right)} + \left(\frac{1}{r}\right) \cos \psi$$

If $\sin(\psi + \eta) > 0$, the satellite is illuminated.

If the satellite is illuminated, calculate

$$\dot{x}'_{RP} = \left(\frac{F_\odot}{M}\right) L_{x\odot}$$

$$\dot{y}'_{RP} = \left(\frac{F_\odot}{M}\right) L_{y\odot}$$

$$\dot{z}'_{RP} = \left(\frac{F_\odot}{M}\right) L_{z\odot}$$

e. Derivatives and Integration

The total perturbative acceleration can now be used to determine the perturbative derivatives of the parameters.

$$\underline{\dot{r}}' = \underline{\dot{r}}'_B + \underline{\dot{r}}'_D + \underline{\dot{r}}'_{RP}$$

$$r\dot{r}' = \underline{r} \cdot \underline{\dot{r}}'$$

$$\dot{s}\dot{s}' = \underline{\dot{r}} \cdot \underline{\dot{r}}'$$

$$r\dot{r} = \underline{r} \cdot \underline{\dot{r}}$$

$$D = \frac{r\dot{r}}{\sqrt{\mu}}$$

$$D' = \frac{r\dot{r}'}{\sqrt{\mu}}$$

$$\dot{D}' = \frac{2\dot{s}\dot{s}'}{\sqrt{\mu}}$$

$$r\dot{b}' = \underline{W} \cdot \underline{\dot{r}}'$$

$$\ell' = \frac{z(r\dot{b}')}{(1+W_z)\sqrt{\mu p}}$$

$$\underline{a}' = \frac{(\dot{D}'\underline{r} - D\dot{\underline{r}}' - D'\underline{r})}{\sqrt{\mu}}$$

$$e\underline{Q} = \underline{W} \times \underline{a}$$

$$-e^2 v = e\underline{Q} \cdot \underline{a}$$

$$L' = \ell' - \frac{2D'}{\sqrt{a}} - \left[\frac{e^2 v'}{1 + \sqrt{1 - e^2}}\right]$$

$$\underline{h}' = \frac{\underline{r} \times \dot{\underline{r}}'}{\sqrt{\mu}}$$

The derivatives, as used in the Runge-Kutta routine[3], are:

$$\frac{dL}{dt} = k_e L' + n$$

$$\frac{d\underline{a}}{dt} = k_e \underline{a}'$$

$$\frac{d\underline{h}}{dt} = k_e \underline{h}'$$

3.3 WEIGHTED DIFFERENTIAL CORRECTION

It is necessary to simulate observations in two phases of the SPWDC. The most obvious phase is that of prediction. Also, in the differential correction, the estimate of the observed quantities on the basis of the current parameter estimates must be obtained. This estimate is often referred to as the "computed" quantity as distinguished from the "observed" quantity. The difference between these quantities is the "residual," e.g., for the ith range observation

$$\Delta\rho_i = \rho_i \text{ (observed)} - \rho_i \text{ (computed)}$$

Each residual is related to corrections in the parameters by an "equation of condition," e.g.,

$$\Delta\rho_i = \sum_{j=1}^{7} \frac{\partial\rho i}{\partial p_j} \Delta p_j$$

where p_j are the parameters n_o, a_{xN_o}, etc.

These equations of condition are weighted, according to the numbers obtained as outlined in Section 3.1, and processed as described in the present section by relating the "computed" quantities to the geocentric ephemeris obtained as described in Section 3.2. This relationship involves the location of the center of the Earth with respect to the observing station, $\underline{R}$:

$$\underline{\rho} = \underline{r} + \underline{R}$$

The partial derivatives with respect to the seven parameters are computed according to analytical formulas, given in the following subsections. When the eccentricity exceeds a given maximum, however, the coefficients of the ballistic parameter are computed by a variant ephemeris. During this calculation the quantities ρ_v, $\underline{L}_v$, and $\dot{\rho}_v$ are stored in a bloc designated VDATA. The v subscript is used hereafter to identify these results of the variant calculation. The variant ephemeris differs from the nominal ephemeris by using a ballistic parameter differing from the nominal value. This variation in the ballistic parameter $\delta\left(\frac{C_D A}{m}\right)$ is represented in the following

formulation by its factor $\delta(\frac{1}{m})$. Therefore, the factor $C_D A$ appears in the coefficients. The correction actually should be thought of as applying to not only the whole ballistic parameter but including a correction to the mean density as well. The correlations between any separate determination of these factors would be very high indeed when only satellite position and velocity observations are used.

a. Preliminary Calculations

This part of the program calculates "observations" of the orbit specified by the input set of elements (or subsequently corrected elements) made from the sensor station. Given the station coordinates of latitude, ϕ, east longitude, λ_E, and height above sea level, H, and Greenwich sidereal time, θ_{to}, at satellite epoch, t_o, the following procedure computes the observations ρ_c, $\dot{\rho}_c$ and the direction cosines of the unit vector, $\underline{L}$, from the station to the satellite for time, t, in minutes since epoch.

Compute local sidereal time, θ, at time, t:

$$\theta = \lambda_E + \theta_{to} + (0.0043752691)\,t \qquad (\text{Mod } 2\pi)$$

Compute the station vector, $\underline{R}$:

$$X = \left[\frac{X}{\cos\ \theta}\right] \cos\theta$$

$$Y = \left[\frac{Y}{\sin\ \theta}\right] \sin\theta$$

$$Z = Z$$

Compute the slant range, ρ_c:

$$\underline{\rho}_c = \underline{r} + \underline{R}$$

$$\rho_c = \sqrt{\rho_x^{\ 2} + \rho_y^{\ 2} + \rho_z^{\ 2}}$$

$$\rho_x = x + X$$

$$\rho_y = y + Y$$

$$\rho_z = z + Z$$

where x, y, z are components of $\underline{r}$ obtained from Section 3.2 by means of input elements and the time, t.

Compute unit vector from the station to the satellite in the equatorial coordinate system:

$$L_{xc} = \frac{\rho_x}{\rho_c}$$

$$L_{yc} = \frac{\rho_y}{\rho_c}$$

$$L_{zc} = \frac{\rho_z}{\rho_c}$$

Compute range rate, $\dot{\rho}_c$,

$$\dot{X} = -Y\,\dot{\theta}, \quad \dot{\theta} = 0.058,834,47$$

$$\dot{Y} = X\,\dot{\theta}$$

$$\dot{Z} = 0$$

$$\underline{\dot{\rho}}_c = \underline{\dot{r}} + \underline{\dot{R}},$$

$$\dot{\rho}_c = \underline{L} \cdot \underline{\dot{\rho}}_c = L_x\,(\dot{x} + \dot{X}) + L_y\,(\dot{y} + \dot{Y}) + L_z\dot{z},$$

where $\dot{x}$, $\dot{y}$, $\dot{z}$ are components of $\underline{\dot{r}}$, obtained from Section 3.2.

b. Range Observations

If range is observed, the residual is

$$R_1 = \rho - \rho_c$$

c. Angular Observations

If azimuth, A, and elevation, h, are observed, the residuals are, respectively,

$$R_2 = \rho_c \underline{\tilde{A}} \cdot (\underline{L} - \underline{L}_c)$$

$$R_3 = \rho_c \underline{\tilde{D}} \cdot (\underline{L} - \underline{L}_c)$$

where

$$\underline{\tilde{A}} = \tilde{A}_{xh} \underline{S} + \tilde{A}_{yh} \underline{E} + \tilde{A}_{zh} \underline{Z}$$

$$\underline{\tilde{D}} = \tilde{D}_{xh} \underline{S} + \tilde{D}_{yh} \underline{E} + \tilde{D}_{zh} \underline{Z}$$

$$\underline{L} = L_{xh} \underline{S} + L_{yh} \underline{E} + L_{zh} \underline{Z}$$

The $\underline{S}$, $\underline{E}$, $\underline{Z}$ unit vector system and the horizon oriented $\underline{L}_h$, $\underline{\tilde{A}}_h$, $\underline{\tilde{D}}_h$ unit vector system are defined by:

$$\underline{S} \begin{cases} S_x = \sin\phi \cos\theta \\ S_y = \sin\phi \sin\theta \\ S_z = -\cos\phi \end{cases}$$

$$
\underline{E} \begin{cases} E_x = -\sin\theta \\ E_y = \cos\theta \\ E_z = 0 \end{cases}
$$

$$
\underline{Z} \begin{cases} Z_x = \cos\phi\cos\theta \\ Z_y = \cos\phi\sin\theta \\ Z_z = \sin\phi \end{cases}
$$

$$
\underline{L}_h \begin{cases} L_{xh} = -\cos A\cos h \\ L_{yh} = \sin A\cos h \\ L_{zh} = \sin h \end{cases}
$$

$$
\underline{\tilde{A}}_h \begin{cases} \tilde{A}_{xh} = \sin A \\ \tilde{A}_{yh} = \cos A \\ \tilde{A}_{zh} = 0 \end{cases}
$$

$$
\underline{\tilde{D}}_h \begin{cases} \tilde{D}_{xh} = \cos A\sin h \\ \tilde{D}_{yh} = -\sin A\sin h \\ \tilde{D}_{zh} = \cos h \end{cases}
$$

If right ascension, α, and declination, δ, are observed, the residuals are:

$$R_4 = \rho_c \underline{A} \cdot (\underline{L} - \underline{L}_c)$$

$$R_5 = \rho_c \underline{D} \cdot (\underline{L} - \underline{L}_c)$$

where

$$\underline{A} \begin{cases} A_x = -\sin \alpha \\ A_y = \cos \alpha \\ A_z = 0 \end{cases}$$

$$\underline{D} \begin{cases} D_x = -\sin \delta \cos \alpha \\ D_y = -\sin \alpha \sin \delta \\ D_z = \cos \delta \end{cases}$$

$$\underline{L} \begin{cases} L_x = \cos \delta \cos \alpha \\ L_y = \cos \delta \sin \alpha \\ L_z = \sin \delta \end{cases}$$

See Figure 4 for these vector relationships.

d. Range Rate Observations

If range rate, $\dot{\rho}$, is observed,

$$R_6 = \rho_c \Delta \dot{\rho} = (\dot{\rho} - \dot{\rho}_c) \rho_c$$

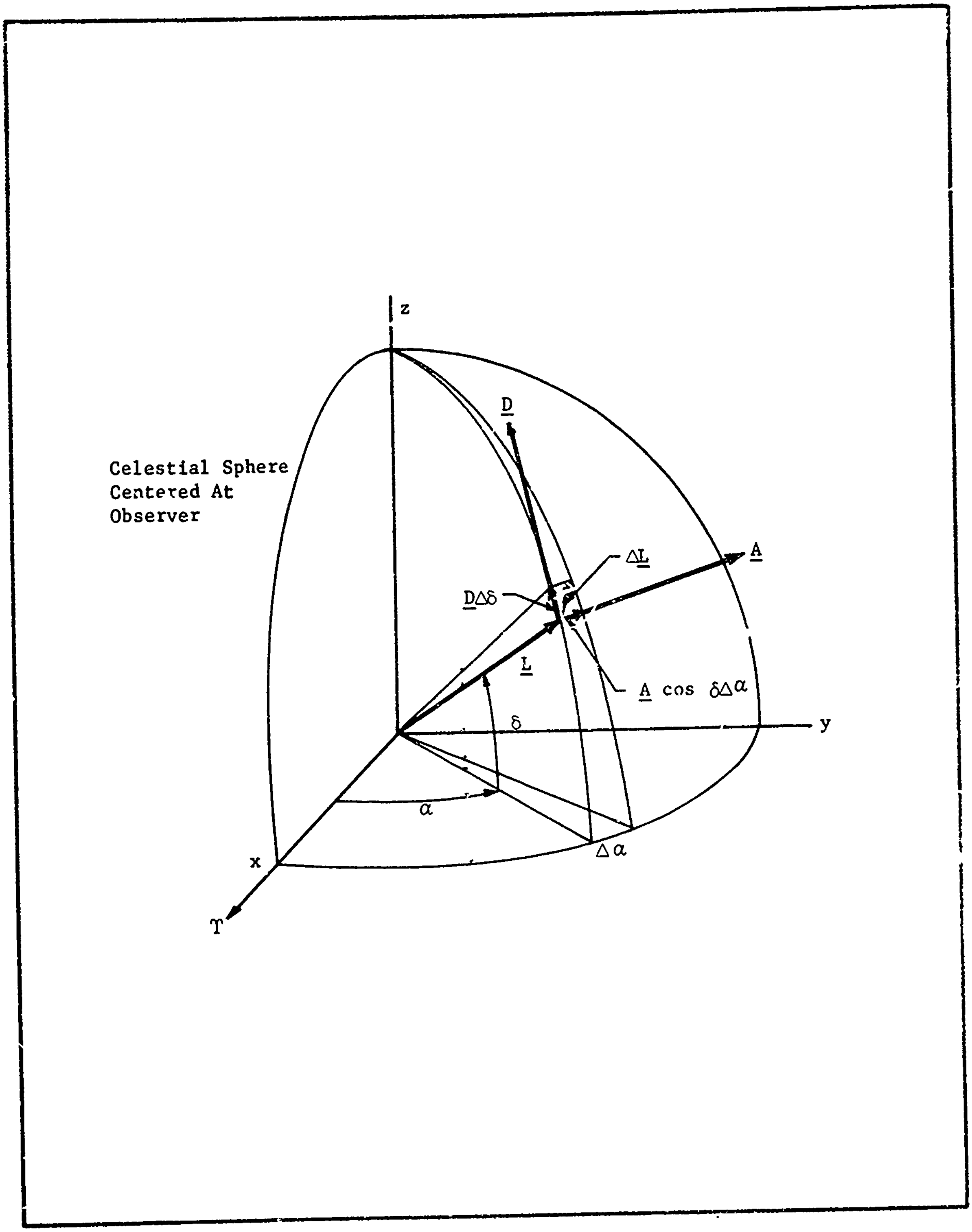


FIG. 4 VECTOR RELATIONSHIPS

e. Differential Correction Solution

The residuals in the observations are related to the changes in the elements through a first order approximation of the form

$$R_i = \left(C_{\frac{\Delta n}{n}}\right)_i \frac{\Delta n_o}{n_o} + \left(C_{\Delta a_{xn}}\right)_i \Delta a_{xn_o} + \left(C_{\Delta a_{yn}}\right)_i \Delta a_{yn_o} + \left(C_{\Delta U_o}\right)_i \Delta U_o + \left(C_{\Delta \Omega}\right)_i \Delta \Omega_o + \left(C_{\Delta i}\right)_i \Delta i_o + \left(C_{\Delta \frac{1}{m}}\right)_i \Delta \frac{1}{m_o}$$

where the form of the coefficients, C_i, depend on the observation type in residual, R_i; time of observation, and the observation weights. These coefficients, which have been developed by means of first-order partials, are functions of the orbit elements and computed observations.

The coefficients are computed by first establishing the R and U coefficients at time t.

$$R_u = (a^2/r)\ e \sin E$$

$$R_n = -2/3\ r + (U - U_o)\ R_u$$

$$R_{xN} = (a^2/r)\left[a_{xN} - \cos (E + \omega)\right]$$

$$R_{yN} = (a^2/r)\left[a_{yN} - \sin (E + \omega)\right]$$

$$U_u = (a^2/r)\sqrt{1 - e^2}$$

$$U_n = (U - U_o)\ U_u$$

$$U_{xN} = \frac{a^2}{r}\left\{(1 + \frac{r}{a}) \sin (E + \omega) + \right.$$

$$\left. a_{xN}\, e \sin E \left[\frac{e^2 - (1 + \sqrt{1-e^2})\, e \cos E}{(1 + \sqrt{1-e^2})^2 \sqrt{1-e^2}}\right] - \frac{a_{yN}}{1 + \sqrt{1-e^2}}\right\}$$

$$U_{yN} = \frac{a^2}{r}\left\{- (1 + \frac{r}{a}) \cos (E + \omega) + \right.$$

$$\left. a_{yN}\, e \sin E \left[\frac{e^2 - (1 + \sqrt{1-e^2})\, e \cos E}{(1 + \sqrt{1-e^2})^2 \sqrt{1-e^2}}\right] + \frac{a_{xN}}{1 + \sqrt{1-e^2}}\right\}$$

When the slant range is observed, the residual and coefficients are given by the expressions:

$$R = \frac{R_1}{\sigma_\rho}$$

$$C_{\frac{\Delta n}{n}} = \left[\underline{L}_c \cdot \underline{U}\, R_n + \underline{L}_c \cdot \underline{V}\, U_n\right] \frac{1}{\sigma_\rho}$$

$$C_{\Delta a_{xN}} = \left[\underline{L}_c \cdot \underline{U}\, R_{xN} + \underline{L}_c \cdot \underline{V}\, U_{xN}\right] \frac{1}{\sigma_\rho}$$

$$C_{\Delta a_{yN}} = \left[\underline{L}_c \cdot \underline{U}\, R_{yN} + \underline{L}_c \cdot \underline{V}\, U_{yN}\right] \frac{1}{\sigma_\rho}$$

$$C_{\Delta U_o} = \left[\underline{L}_c \cdot \underline{U}\, R_u + \underline{L}_c \cdot \underline{V}\, U_u\right] \frac{1}{\sigma_\rho}$$

$$C_{\Delta\Omega} = \left[\underline{L}_c \cdot \underline{V}\, r \cos i - \underline{L}_c \cdot \underline{W}\, r \sin i \cos u\right] \frac{1}{\sigma_\rho}$$

$$C_{\Delta i} = \left[\underline{L}_c \cdot \underline{W}\, r \sin u\right] \frac{1}{\sigma_\rho}$$

$$C_{\Delta \frac{1}{m}} = \left[\frac{\rho_v - \rho_c}{\delta (\frac{1}{m})}\right] \frac{1}{\sigma_\rho}$$ where the v subscript refers to

the range computed from the variant ephemeris. If computed by the analytic formula,

$$C_{\Delta \frac{1}{m}} = \left[\frac{2}{3}\, a\, n\, (t - t_o)\, C_D\, A \rho_r\, C_{\frac{\Delta n}{n}}\, K\right] \frac{1}{\sigma_\rho}$$

where K is a unit conversion constant and C_D is set equal to 2. The quantity σ_ρ in the coefficient equations is the standard deviation in the range measurement, so that $\frac{1}{\sigma_\rho}$ is the weight of that measurement, assigned as described by Section 3.1.

When azimuth A is observed, then $R = \frac{R_2}{\sigma_A}$ and the coefficients are obtained by replacing $\underline{L}$ by $\underline{A}$. In this case, the $\frac{1}{\sigma_\rho}$ is replaced by $\frac{1}{\rho_c \sigma_A}$ where σ_A is the standard deviation in the azimuth measurement. The seventh coefficient, in the variant ephemeris mode, must be computed as

$$C_{\Delta \frac{1}{m}} = \frac{1}{\rho_c \sigma_A} \left[\frac{\rho_c \underline{A} \cdot (\underline{L}_v - \underline{L}_c)}{\delta (\frac{1}{m})} \right]$$

With the azimuth, elevation h is always observed, and in this case $R = \frac{R_3}{\sigma_h}$ and the corresponding coefficients are obtained by replacing $\underline{L}$ with $\underline{D}$. $\frac{1}{\sigma_\rho}$ is replaced by $\frac{1}{\rho_c \sigma_h}$, where σ_h is the standard deviation in the elevation measurement. The seventh coefficient in the variant ephemeris mode is

$$C_{\Delta \frac{1}{m}} = \frac{1}{\rho_c \sigma_h} \left[\frac{\rho_c \tilde{\underline{D}} \cdot (\underline{L}_v - \underline{L}_c)}{\delta (\frac{1}{m})} . \right]$$

If right ascension α is observed, then $R = \frac{R_4}{\sigma_\alpha}$ and the corresponding coefficients are obtained by replacing $\underline{L}$ with $\underline{A}$. $\frac{1}{\sigma_\rho}$ is replaced by $\frac{1}{\rho_c \sigma_\alpha}$, where σ_α is the standard deviation in the right ascension. The seventh coefficient in the variant ephemeris mode is

$$C_{\Delta\frac{1}{m}} = \frac{1}{\rho_c \sigma_\alpha} \left[\frac{\rho_c \underline{A} \cdot (\underline{L}_v - \underline{L}_c)}{\delta (\frac{1}{m})} \right]$$

Along with right ascension, declination δ is always observed, and in this case $R = \frac{R_5}{\sigma_\delta}$ and the corresponding coefficients are obtained by replacing $\underline{L}_c$ with $\underline{D}$. $\frac{1}{\sigma_\rho}$ is replaced by $\frac{1}{\rho_c \sigma_\delta}$, where σ_δ is the standard deviation in the declination. The seventh coefficient in the variant ephemeris mode is

$$C_{\Delta\frac{1}{m}} = \frac{1}{\rho_c \sigma_\delta} \left[\frac{\rho_c \underline{D} \cdot (\underline{L}_v - \underline{L}_c)}{\delta (\frac{1}{m})} \right]$$

If range rate, $\dot{\rho}$, is observed, the preliminary coefficients are computed from:

$$\dot{R}_u = \sqrt{\mu}\, \frac{a^{5/2}}{r^3} (e \cos E - e^2)$$

$$\dot{R}_n = \frac{\dot{r}}{3} + (U - U_o)\, \dot{R}_u$$

$$\dot{R}_{xN} = \sqrt{\mu}\, \frac{a^{5/2}}{r^3} [\sin (E + \omega) - a_{xN}\, e \sin E - a_{yN}]$$

$$\dot{R}_{yN} = \sqrt{\mu}\frac{a^{5/2}}{r^3}\left[- \cos (E+\omega) - a_{yN}\, e \sin E + a_{xN}\right]$$

$$\dot{U}_u = -\sqrt{\mu}\,\frac{a^{5/2}}{r^3}\sqrt{1 - e^2}\; e \sin E$$

$$\dot{U}_n = \frac{r\dot{v}}{3} + (U - U_o)\,\dot{U}_u$$

$$\dot{U}_{xN} = \sqrt{\mu}\frac{a^{5/2}}{r^3}\sqrt{1 - e^2}\left[\cos (E+\omega) - a_{xN}\left(1 + \frac{r^2}{ap}\right)\right]$$

$$\dot{U}_{yN} = \sqrt{\mu}\frac{a^{5/2}}{r^3}\sqrt{1 - e^2}\left[\sin (E+\omega) - a_{yN}\left(1 + \frac{r^2}{ap}\right)\right]$$

The range rate coefficients are then computed as

$$C_{\frac{\Delta n}{n}} = \Big\{ \underline{L}_c \cdot \underline{U}\,[\rho_c\,(\dot{R}_n - \dot{v}U_n) - \dot{\rho}_c\, R_n] + \dot{\underline{\rho}}_c \cdot \underline{U}\, R_n$$

$$+ \underline{L}_c \cdot \underline{V}\,[\rho_c\,(\dot{U}_n + \frac{\dot{r}}{r}\, U_n) - \dot{\rho}_c\, U_n] + \dot{\underline{\rho}}_c \cdot \underline{V}\, U_n \Big\} \frac{1}{\sigma_{\dot{\rho}}}$$

$$C_{\Delta a_{xN}} = \Big\{ \underline{L}_c \cdot \underline{U}\,[\rho_c\,(\dot{R}_{xN} - \dot{V}\, U_{xN}) - \dot{\rho}_c\, R_{xN}] + \dot{\underline{\rho}}_c \cdot \underline{U}\, R_{xN}$$

$$+ \underline{L}_c \cdot \underline{V}\,[\rho_c\,(\dot{U}_{xN} + \frac{\dot{r}}{r}\, U_{xN} - \dot{\rho}_c\, U_{xN}] + \dot{\underline{\rho}}_c \cdot \underline{V}\, U_{xN} \Big\} \frac{1}{\sigma_{\dot{\rho}}}$$

$$C_{\Delta a_{yN}} = \Big\{ \underline{L}_c \cdot \underline{U}\,[\rho_c\,(\dot{R}_{yN} - \dot{V}\, U_{yn} - \dot{\rho}_c\, R_{yN}] + \dot{\underline{\rho}}_c \cdot \underline{U}\, R_{yN}$$

$$+ \underline{L}_c \cdot \underline{V}\,[\rho_c\,(\dot{U}_{yN} + \frac{\dot{r}}{r}\, U_{yN}) - \dot{\rho}_c\, U_{yN}] + \dot{\underline{\rho}}_c \cdot \underline{V}\, U_{yN} \Big\} \frac{1}{\sigma_{\dot{\rho}}}$$

$$C_{\Delta U_o} = \Big\{ \underline{L}_c \cdot \underline{U}\,[\rho_c\,(\dot{R}_u - \dot{v}\, U_u) - \dot{\rho}_c\, R_u] + \dot{\underline{\rho}}_c \cdot \underline{U}\, R_u$$

$$+ \underline{L}_c \cdot \underline{V}\,[\rho_c\,(U_u + \frac{\dot{r}}{r}\, U_u) - \dot{\rho}_c\, U_u + \dot{\underline{\rho}}_c \cdot \underline{V}\, U_u \Big\} \frac{1}{\sigma_{\dot{\rho}}}$$

$$C_{\Delta\Omega} = \Big\{ - \rho_c \underline{L}_c \cdot \underline{U}\, r\, \dot{v} \cos i + \underline{L}_c \cdot \underline{V} \cos i\, [\, \rho_c\, \dot{r} - \dot{\rho}_c\, r\,]$$

$$+ \underline{\dot{\rho}}_c \cdot \underline{V}\, r \cos i + \underline{L}_c \cdot \underline{W} \sin i\, [\rho_c\, (r\, \dot{v} \sin u - \dot{r} \cos u$$

$$+ \dot{\rho}_c\, r \cos u\,] - \underline{\dot{\rho}}_c \cdot \underline{W}\, r \sin i \cos u \Big\} \frac{1}{\sigma_{\dot{\rho}}}$$

$$C_{\Delta i} = \Big\{ \underline{L}_c \cdot \underline{W}\, [\rho_c\, (r\, \dot{v} \cos u + r \sin u) - \dot{\rho}_c\, r \sin u\,]$$

$$+ \underline{\dot{\rho}}_c \cdot \underline{W}\, r \sin u \Big\} \frac{1}{\sigma_{\dot{\rho}}}$$

$$C_{\Delta \frac{1}{m}} = \left\{ \frac{\rho_c (\dot{\rho}_v - \dot{\rho}_c)}{\delta\, (\frac{1}{m})} \right\} \frac{1}{\sigma_{\dot{\rho}}}$$

If computed by the analytic formula,

$$C_{\Delta \frac{1}{m}} = [\, \frac{2}{3}\, a\, n\, (t - t_o)\; C_D\, A\, \rho_r\, C_{\frac{\Delta n}{n}}\, K\,] \frac{1}{\sigma_{\dot{\rho}}}$$

Let $\quad \sum_{j=1}^{N} C_{ij}\, \Delta_j = R_{ij}$

represent all such equations of condition, where C_{ij} are the coefficients, R_{ij} are the accepted observation residuals, Δ_j are the corrections to the orbital elements, and N is the number of elements to be corrected and is the number of accepted observation residuals. The resulting matrix equation is solved to give the corrections, Δ_j in a least square sense, to the orbital elements at time, t_o. These corrections are applied as follows (primes denote corrected elements):

$$n'_o = n_o \left(1 + \frac{\Delta n_o}{n_o}\right)$$

$$U'_o = U_o + \Delta U_o$$

$$a'_{xNo} = a_{xN_o} + \Delta a_{xN}$$

$$a'_{yNo} = a_{yNo} + \Delta a_{yN}$$

$$\Omega'_o = \Omega_o + \Delta \Omega$$

$$i'_o = i_o + \Delta i$$

$$\begin{cases} L'_o = U'_o + \Omega'_o \text{ if } W'_z = \cos i \geq 0 \\ L'_o = U'_o - \Omega'_o \text{ if } W'_z = \cos i < 0 \end{cases}$$

Following the above calculation of the corrected elements, another representation of the observations is performed, on the basis of the corrected elements, and another set of residuals is formed by using the same input observations. The RMS values of the last two sets of consecutive residuals are compared to insure convergence of the computational process. The process is complete when the residual RMS converges to the minimum value considered as acceptable.

3.4 PREDICTION AND RELIABILITY

Once a corrected set of elements has been established, the Special Perturbations formulation is used to compute future position and velocity. The accuracy of the prediction may be estimated by means of an analysis of the effect of the element uncertainties by their propagation through the representation equations.

a. Prediction Options

The prediction function applies the equations of Section 3.2 to represent the position and velocity at some specified time. Two modes of prediction have been formulated: (1) prediction by specifying the initial time, the number of points and some time interval and (2) prediction for the time of closest approach to some specified sensor.

Prediction by time is a process of representing the position and velocity at the requested times. In the Special Perturbations formulation this requires an integration to the initial time point followed by integration to the subsequent time points, as specified by the time interval, until the final time is reached.

Prediction by sensor requires a more complex logic. Here, the numerical integration proceeds until the range rate relative to the station changes sign from negative to positive. At this point, the integration is used as part of an iterative scheme to obtain the time when range rate is zero. Once established, this is considered to be the time of closest approach. The routine then goes on to obtain prediction points for specified time intervals and duration on either side of the time of closest approach. This process may be repeated for any number of passes over the sensor; however, it is restricted to one sensor.

b. Reliability Estimates

The prediction function has the capability of estimating the reliability of its prediction points. This estimate is obtained by means of the variance-covariance matrix of orbit elements established in the final iteration in the differential correction process (Section 3.3 e). This matrix may be a six by six or a seven by seven depending on the nature of the differential correction. In either case the variance-covariance matrix may be defined as $[A^{-1}]$, and the quadratic form

$$[D] = [G][A^{-1}][G]^T$$

leads to the variance-covariance matrix of predicted position $[D]$.

The variance-covariance matrix of predicted velocity $[\dot{D}]$ is obtained from the quadratic form

$$[\dot{D}] = [H][A^{-1}][H]^T$$

The formulation for the standard deviation in predicted position and predicted velocity requires the formation of the $[G]$ and $[H]$ matrices. The $[G]$ matrix is used to relate uncertainties in orbital elements to uncertainties in predicted position. The $[H]$ matrix is used to relate uncertainties in orbital elements to uncertainties in predicted velocity.

The $[G]$ matrix is composed of the following matrix elements:

$$g_{11} = R_n$$

$$g_{12} = R_{xn}$$

$$g_{13} = R_{yn}$$

$$g_{14} = R_u$$

$$g_{15} = 0$$

$$g_{16} = 0$$

$$g_{17} = 2/3\ an(t-t_o)\ C_D\ A\ \rho_r\ R_n$$

$$g_{21} = U_n$$

$$g_{22} = U_{xn}$$

$$g_{23} = U_{yn}$$

$$g_{24} = U_u$$

$$g_{25} = r \cos i$$

$$g_{26} = 0$$

$$g_{27} = 2/3\ an(t-t_o)\ C_D\ A\ \rho_r\ U_n$$

$$g_{31} = 0$$

$$g_{32} = 0$$

$$g_{33} = 0$$

$$g_{34} = 0$$

$$g_{35} = -r \sin i \cos u$$

$$g_{36} = r \sin u$$

$$g_{37} = 0$$

The $[H]$ matrix is composed of the following matrix elements:

$$h_{11} = (\dot{R}_n - \dot{v}\ U_n)$$

$$h_{12} = \dot{R}_{xn} - \dot{v}\ U_{xn}$$

$$h_{13} = \dot{R}_{yn} - \dot{v}\ U_{yn}$$

$$h_{14} = \dot{R}_u - \dot{v}\ U_u$$

$$h_{15} = -\ r\dot{v} \cos i$$

$$h_{16} = 0$$

$$h_{17} = 2/3\ an(t-t_o)\ C_D\ A\ \rho_r\ (\dot{R}_n - \dot{v}U_n)$$

$$h_{21} = \dot{U}_n + \frac{\dot{r}}{r} U_n$$

$$h_{22} = \dot{U}_{xn} + \frac{\dot{r}}{r} U_{xn}$$

$$h_{23} = \dot{U}_{yn} + \frac{\dot{r}}{r} U_{yn}$$

$$h_{24} = \dot{U}_u + \frac{\dot{r}}{r} U_u = 0$$

$$h_{25} = \dot{r} \cos i$$

$$h_{26} = 0$$

$$h_{27} = 2/3\ an(t-t_o)\ C_D\ A\ \rho_r\ (\dot{U}_n + \frac{\dot{r}}{r} U_n)$$

$$h_{31} = 0$$

$$h_{32} = 0$$

$$h_{33} = 0$$

$$h_{34} = 0$$

$$h_{35} = (r\dot{v} \sin u - \dot{r} \cos u) \sin i$$

$$h_{36} = r\dot{v} \cos u + \dot{r} \sin u$$

$$h_{37} = 0$$

The quantities R_n, R_{xn}, R_{yn}, R_u, U_n, U_{xn}, U_{yn}, U_u, $\dot{R}_n$, $\dot{R}_{xn}$, $\dot{R}_{yn}$, $\dot{R}_u$, $\dot{U}_u$, $\dot{U}_{xn}$, $\dot{U}_{yn}$, $\dot{U}_u$ are defined in Section 3.3 e.

The matrices $[G]$ and $[H]$ are functions of the orbit parameter and the prediction time. $[G]^T$ and $[H]^T$ are the matrix transpose of $[G]$ and $[H]$, respectively.

Thus, the reliability estimates are obtained from the main diagonals of $[D]$ and $[\dot{D}]$. These estimates are the standard deviations of the in-track, cross-track, and out-of-plane components of position and velocity at the prediction time.

SECTION 4

OPERATING INSTRUCTIONS

The Executive Program of the SPS B-2 System processes the Element, Sensor, Observation and Parameter cards leaving the data in specified B-2 core buffer locations to be accessed by OBSWGT and SPWDC.

The Observation Weighting Program will reformat the observations, ordering them in time and applying weights (optional) for a maximum of 400 observations. It then calls SPWDC for execution.

The SPWDC program may go through a differential correction process, prediction by station, or prediction by time as specified on the Program Parameter Cards.

4.1 TAPE SETUP

The logical tape assignments are:

Logical Unit	Tape Description
1	SPS B-2 Binary Master
2	Input
4	SEAI
6	SRADU (optional observation input)
7	Weight tape (optional)
11	Output
12	$\underline{\dot{r}}$, $\underline{\dot{r}}$ binary output tape (optional)

4.2 DECK SETUP

Input to the SPWDC Program consists of SPS control cards, Parameter cards, Element cards, Sensor cards and Observation cards (Figure 5).

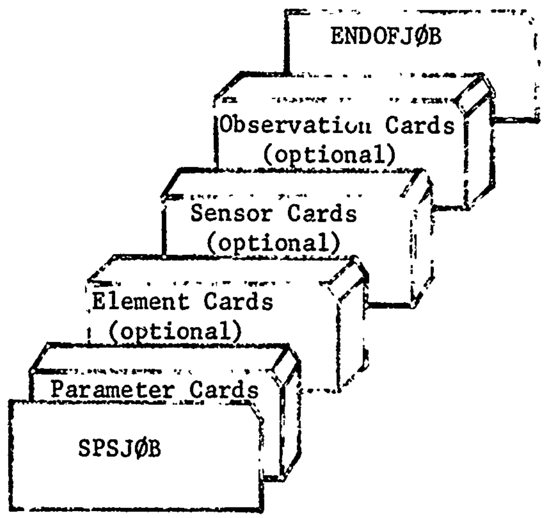


FIGURE 5. SPWDC INPUT DECK

4.3 INPUT

a. Program Notes

(1) The input options for the SPSJOB card are to be found in Appendix I.

(2) The differential correction is controlled by the following cards:

(a) The OBSWGT Parameter card calls the SPWDC Program.

(b) The Vehicle Characteristic card contains the mass, diameter, and reflectivity of the satellite for

the drag and radiation pressure computations. This card may be omitted if the following values are acceptable:

mass = 10.0 kg

diameter = 1.0 meter

reflectivity = 1.0

(c) The Differential Correction Control card contains the pertinent information for the integration and differential correction.

(d) The Absolute Error Control card may be omitted if

$$a_L = 10^{-7}$$

$$a_{\underline{a}},\ a_{\underline{h}} = 10^{-8}$$

are acceptable values.

(e) The Relative Error Control card may be omitted if

$$r_L,\ r_{\underline{a}},\ r_{\underline{h}} = 0$$

are acceptable values.

(f) The Program Execution card specifies that a differential correction is to be executed and starts operation of the program.

(3) Prediction by time is controlled by the following cards:

(a) OBSWGT Control card

(b) Absolute Error Control card

(c) Relative Error Control card

(d) Time Prediction card #1. (This card contains integration and output information.)

(e) Time Prediction card #2. (The time output interval is indicated on this card.)

(f) Program Execution card. (The Prediction-by-time path is set, and execution begins.)

(4) Prediction-by-station is controlled by these cards:

(a) OBSWGT Parameter card

(b) Absolute Error Control card

(c) Relative Error Control card

(d) Station Pass Prediction card #1. (The integration and output information is specified.)

(e) Station Pass Prediction card #2. (Sensor data may be input from this card or from a B-2 SEAI tape.)

(f) Program Execution card. (The Prediction-by-sensor path is set, and execution begins.)

(5) Each of the three, differential correction, prediction-by-station, or prediction-by-time, may be run separately, or the combination of differential correction and prediction-by-station or time may be executed. For actual test cases and variations of logical paths, see Section 6.

b. Input Deck

A typical input deck is arranged as follows:

70 SCHTP $({}^{11}8_2)$

JOB Card

REM Card

SPSJOB

Parameter Cards

Element Cards (SPADATS format)[3]

Sensor Cards (SPADATS format)[3]

Observation Cards (SPADATS format)[3]

ENDOFJOB $({}^{11}8_2)$

ENDSCHED $({}^{11}8_2)$

Since the Element, Sensor and Observation cards are standard SPADATS System cards, only the Program Parameter cards and the Weighting Tape Input will be described in this Section.

(1) Parameter Card Formats

The first parameter card must be the OBSWGT card which contains the identity of the program called by OBSWGT after processing the observations. The rest of the parameter cards may be in any order, with the exception of the Program Execution card which must be the last card. Note that:

(a) If six or seven elements are being corrected, the n only correction and the Δq check may be chosen as options. (Differential Correction Control card)

(b) The prediction reliance option for prediction-by-time or station is only possible if six or seven elements are to be differentially corrected, the observations are weighted, and the drag perturbation is computed (for correction of seven elements). If these conditions are not met, the flag is turned off and the run proceeds without prediction reliance. (Time and Station Prediction cards)

Appendix I contains the Parameter card formats and related code information.

(2) Weighting Cards

Additional weighting input data may be contained on a weighting tape which has the following card deck setup:

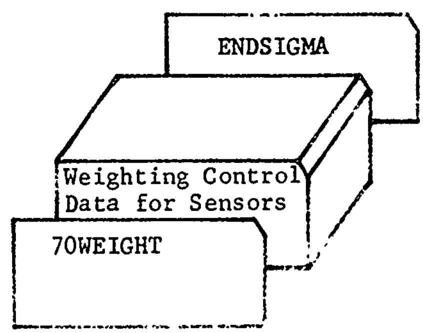


FIGURE 6. CARD DECK FORMAT FOR THE WEIGHTING TAPE

Data will consist of the standard deviations (1) for the observations specified as constants for each sensor, (2) for each observation, or (3) specified by control information for determining the method to be used for computing the standard deviation.

There may be weighting of all observations or no weighting, but in a particular job there cannot be both weighted and unweighted observations. There must be sigmas for all quantities observed or the observation will be rejected for lack of weighting information. Weighting information is obtained from Weight tape or from the file data in the Observation Weighting Program. (Appendix I contains the card formats for the Weighting tape.)

A weighting tape must contain weighting information for (1) sensors only, (2) observations only, (3) functions only, or (4) sensors and functions.

4.4 OUTPUT

The output for SPWDC consists of element cards, printed data, and optionally a binary tape and/or punched cards. The SPADATS seven element cards plus the vehicle characteristic card are punched out (data select 2, code mode) after the successful completion of the differential correction of the orbit. A binary tape containing time in minutes since epoch, position ($\underline{r}$) and velocity ($\dot{\underline{r}}$) is an optional output for prediction. See Figure 7.

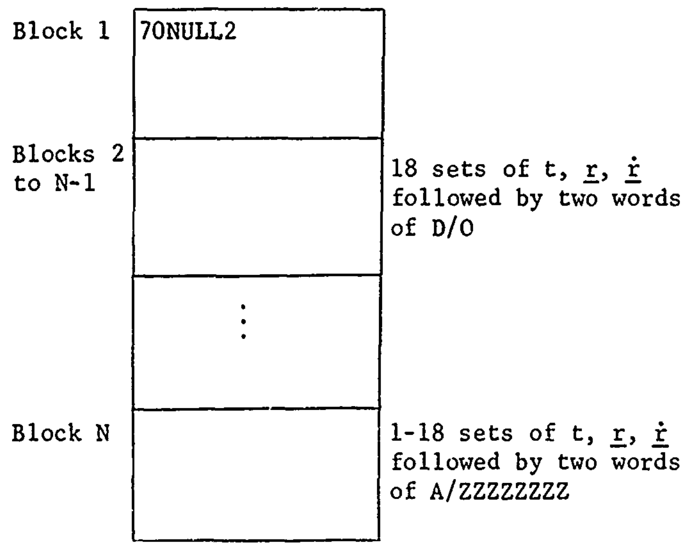


FIGURE 7. CONTENTS OF BINARY TAPE

The printed data (data select 1) consist of heading lines, differential correction output, prediction output and error comments. An additional punched card option will output special format cards containing time and position data. The punched cards contain $\underline{r}$ and integral days since beginning of the year (first card) and fractional time (second card) in the format

ID $\pm$ XXXXXXXXX $\pm$ YY

where ID is an identifier:

ID		
08	X	
09	Y	
10	Z	
11	t	(integral days since beginning of year)
12	t	(fractional day)

a. Initial Output

The first line of each page will be:

SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL
CORRECTION PRG (Date) PAGE__________

This is followed by the card image of the parameter cards. Any error on an input parameter card will cause the following comment to be printed:

ERROR IN INPUT CARD NO__________ JOB TERMINATED

If the proper sensor data was not found on the SEAI tape or on a sensor card, the remark

NO COORDINATES FOR STATION NO________, OBS. SKIPPED

is printed. When no standard deviations are found for an observation,

NO WEIGHTING INFORMATION FOR OBS. FROM STATION
NO_________, OBS. SKIPPED

is printed.

b. Differential Correction Output

The differential correction output consists of the residuals (optionally) for all of the observations, followed by the root-mean-squares, correction to the elements and corrected elements. For a weighted run the normalized root-mean-squares are also output. This output is repeated until the differential correction converges or diverges.

For convergence the final set of residuals is followed by the comment

DC CONVERGED - THE NEXT CORRECTIONS WOULD BE . . .

and the final set of corrections. This is followed by

NO. OF RESIDUALS USED = _______ NO. OF RESIDUALS REJECTED = ________

printed with the values. Then follows

OLD ELEMENTS WITH RESPECT TO OLD EPOCH . . .

OLD ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH . . .

NEW ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH . . .

NEW ELEMENTS WITH RESPECT TO NEW EPOCH . . .

with appropriate values. The seven element cards and the vehicle characteristic card are punched out when the differential correction has converged. Following this output, the comment

ELEMENT SET UPDATED

is printed. For divergence, the error comment

DIFFERENTIAL CORRECTION DID NOT CONVERGE, JOB TERMINATED

is followed by the proposed corrections to the elements and the corrected elements with respect to the old epoch.

c. Prediction Output

For prediction by print triples there are four output options. The time in days since beginning of the year is followed by any or all of the following:

(1) position and velocity ($\underline{r}$, $\dot{\underline{r}}$)

(2) osculating orbital elements (a, e, i, Ω, ω, U)

(3) subsatellite track (ϕ, λ_E, h)

For prediction by station pass there is, in addition, the option to output

(4) acquisition coordinates ($\rho, \dot{\rho}$, A, h)

d. Prediction Reliability Output

The optional prediction reliability output will print the standard deviation in the predicted position and velocity components at each prediction point. The format provides two lines of data per time point. On the first line the standard deviations are referred to the cross-track, in-track and out-of-plane coordinates, and on the second line they are referred to the inertial x, y, z coordinate system.

e. Error Comments

The following are error comments which may appear on the printed output:

(1) An error in the Runge-Kutta integration routine will give one of the following comments:

ERROR IN INTEGRATING VARIANT EPHEMERIS,
JOB TERMINATED. (Correcting Mass)

ERROR IN INTEGRATING EPHEMERIS FOR DC,
JOB TERMINATED. (No Mass Correction)

ERROR IN RUNGE-KUTTA IN PREDICTION BY
TIME RUN TERMINATED. (Prediction by Time)

A(I) + R(I)Y(I)=0 ERROR IN RUNGE KUTTA
INTEGRATION (Prediction by Station)

DELTA T = 0 AT ANY PT IN THE INTEGRATION
PROCESS (Prediction by Station)

(2) In computing the satellite altitude for the drag perturbation for too-low a satellite, this comment is printed:

SATELLITE DROPPED BELOW 50 KMS. TIME
SINCE EPOCH = _________MIN.

(3) When all the observations are rejected before the differential correction,

NO GOOD OBSERVATIONS END OF JOB

is printed.

(4) If Kepler's equation could not be solved for $(E + \omega)$ before entering the integration routine, the comment

DID NOT CONVERGE IN KEPLERS EQN IN 50
ITERATIONS, RUN TERMINATED

appears on the output.

(5) If the number of good observations was less than the number of elements to be corrected,

DC FAILED. NOT ENOUGH GOOD OBSERVED QUANTITIES
TO SOLVE THE LEAST SQUARES MATRIX, JOB TERMINATED

is printed.

(6) If the RMS gets larger twice in a row, if there is a change greater than 0.5% in the RMS, or if the maximum number of passes through the differential correction have been made, the following is printed:

DIFFERENTIAL CORRECTION DID NOT CONVERGE,
JOB TERMINATED.

(7) When the input Δq is greater than the computed Δq, this comment appears:

DELTA Q TOO LARGE, REPEAT CORRECTION WITHOUT
CORRECTION AXN AND AYN.

(8) When the prediction reliability check is attempted in a correction involving less than six elements, this comment appears:

INSUFFICIENT EQUATIONS TO OUTPUT SIGMAS

SECTION 5

FLOW DIAGRAMS

The following flow diagrams display the computational procedures and logical flow used in the program. Standardized symbols are maintained throughout. Ovals are used to indicate subroutines. Rectangles are used for computational processes. Diamonds represent logical decisions or branching tests.

5.1 MAIN FLOW DIAGRAMS

The main flow of the program is shown in macro structure on the following two pages. These diagrams display the differential correction function and the prediction function of SPWDC.

5.2 DETAILED FLOW DIAGRAMS

The detailed flow diagrams are displayed in Appendix II and show all program loops within SPWDC.

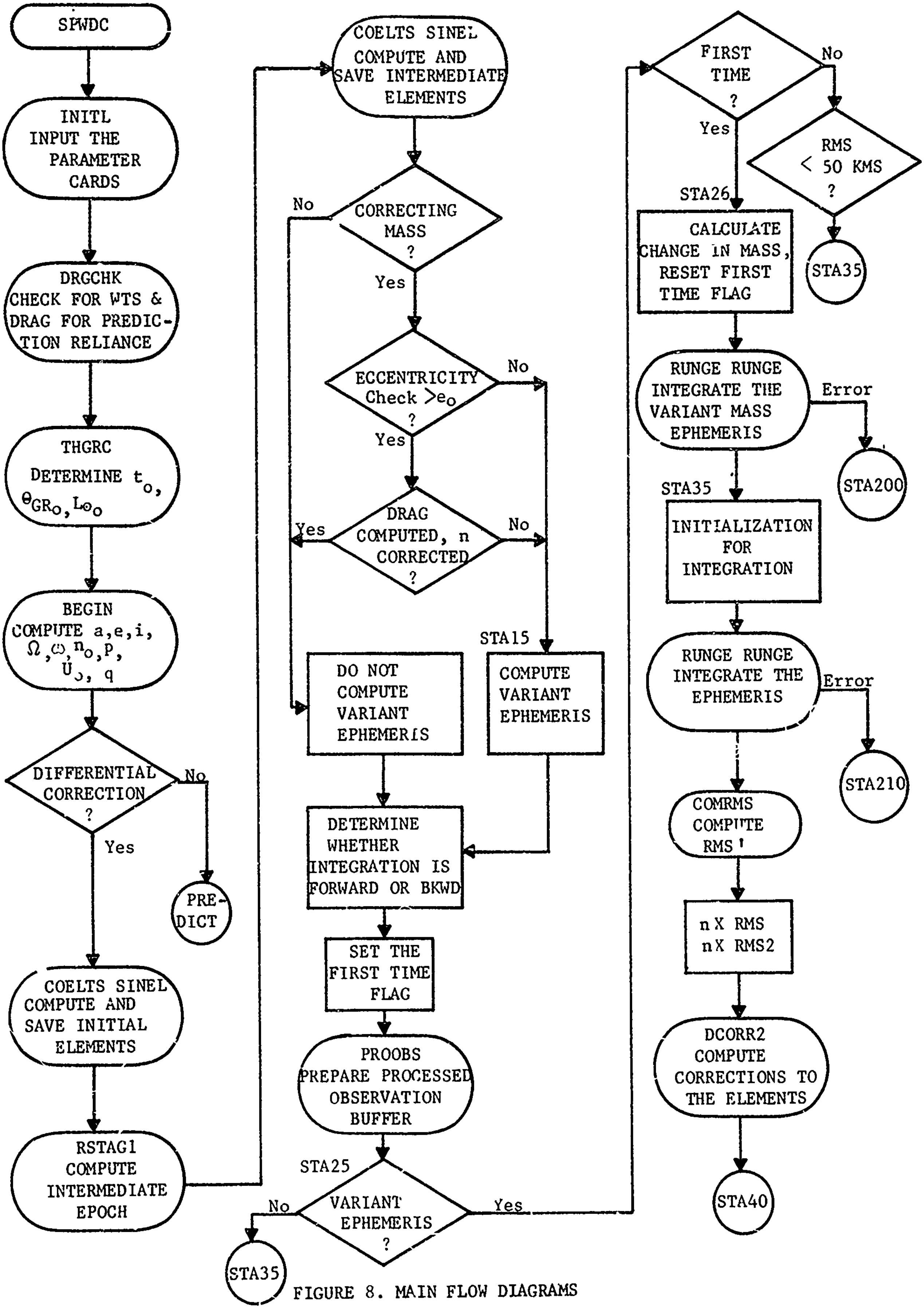


FIGURE 8. MAIN FLOW DIAGRAMS

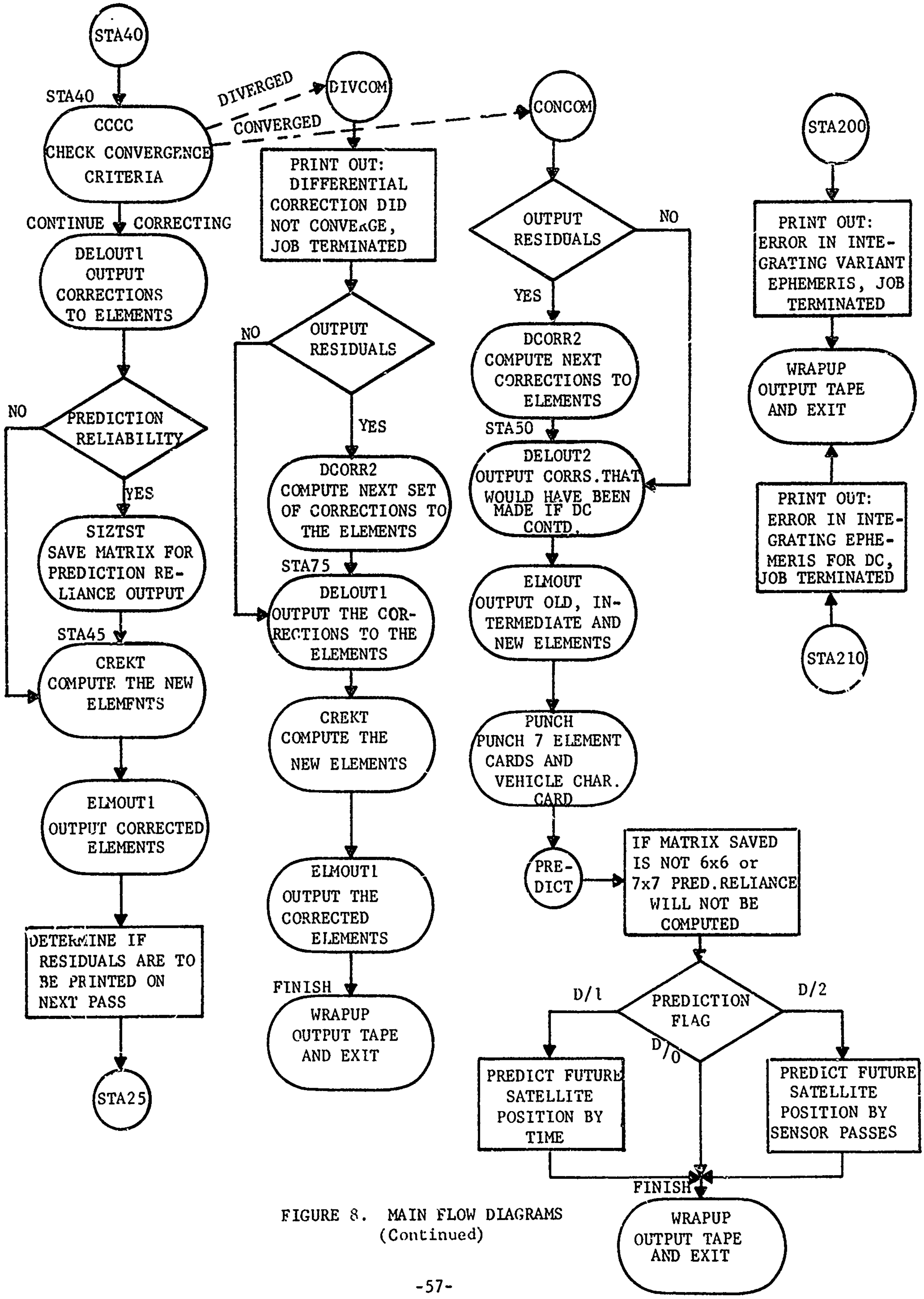


FIGURE 8. MAIN FLOW DIAGRAMS
(Continued)

SECTION 6

COMPILED TEST CASES

The three test cases for Satellite 116 described in this section will test the logical paths of the program. Note that the Observation and Element cards are the same for all cases and the sensors are obtained from the SEAI tape. The vehicle characteristics for 61 OMICRON 1 are diameter 1.1283 m, and mass 79.378 kg.

6.1 TEST CASE 1

See Figure 9, Weighted Differential Correction.

The first test case is a weighted differential correction with 11 observations covering a 40-hour period. The element set and observations are card inputs to the program; the sensors are obtained from the SEAI tape (input option 5). The weighting data are from the Weight Tape (Figure 15). Bulge, drag, and radiation pressure perturbations are included in the variable integration with a starting step size of -1.0 minute. Six elements are to be corrected in a maximum of 9 passes through the differential correction with an n only correction on the first pass.

The absolute maximums for rejection are 1000 km and 0.5 km/sec; the rms multiplier is 1.5. All residuals are to be output, and the new epoch is the 10030 revolution.

See Figure 10 for Test Case 1 output.

6.2 TEST CASE 2

See Figure 11, Weighted Differential Correction and Prediction by Time.

The ephemeris calculation incorporates a variable negative integration with an initial step size of one minute taking into account bulge and drag perturbations and taking the new epoch to be 200.7 days since the beginning of the year. Six elements are to be corrected with an n only correction on the first pass and a Δq check of 500 km. The time prediction integration is in the variable mode with an initial step size of 1.0 minute taking into account bulge and drag perturbations. Ten points are output at a one-minute interval starting at 200.7 days (July 19, 1963). All printed options including prediction reliance values are output as well as the binary tape and punched cards.

See Figure 12 for Test Case 2 output.

6.3 TEST CASE 3

See Figure 13, Unweighted Differential Correction and Prediction by Station.

This is an unweighted differential correction run updating the epoch to the time of the last observation. The integration is in the variable mode with an initial step size of -1.0 minute; bulge and drag perturbations are included. Six elements are differentially corrected with an n only correction on the first pass. The initial rejection maximums are 1000 km and 0.5 km/sec with an rms multiplier of 1.5. There is a maximum of nine passes through the differential correction with the residuals output each time.

In the station prediction the integration mode is variable with a two-minute step size; the Earth's bulge and drag are included. Printed output consists of t; $\underline{r}$, $\dot{\underline{r}}$; a, e, i, Ω, ω, U; $\emptyset$, λ_E, h; and ρ, $\dot{\rho}$, A, h for sensor 216 (coordinates are on the SEAI tape). The output points will be 5 minutes on either side of the point of closest approach at a 1.0 minute interval provided $h > 1.0^{\circ}$.

See Figure 14 for Test Case 3 output.

```
70 SCHTP
                JOB     TEST CASE 1
                REM     SAT 116 40HR OBS DC ONLY-WEIGHTED
SPSJOB  OBSWGTE 50
SPWDC                                                                          P
  TEST CASE 1 SAT 116                     1.1283    79.378   .1.0            2P
 -1.0     111 10030.          111111 91          1000.    .5      1.5    2    3P
1                                                                            10P
116 20 1610MICRON1             10000   .0070763397  66.844600456               E
116 20 21963             200.7000000000              310.611758                E
116 20 3  .0720959506 78.707605     132.86108076      1.138844321              E
116 20 4                -21.419242667   -.703494501                            E
116 20 5 .0000                                                                 E
116 20 6                                                                       E
116 20 7                          31201000000.00                               E
11622003903071804001800945000025490000118898
11625 039030718040308546282 70 19636000161120 457734220             0000
11625 2163307180022194831636 0 35189000212596-519002660             0000
11625 2163307180209246201764 0 30407000 205147-491444840            0000
11625 2163307181351148610700 0 09008000292007 080338390             0000
11625 2163307181534570925664 0 15049000110276-208119980             0000
11625 2163307182103210130246 0 01107000324938 022869110             0000
11625 2163307182255138961019 C 05846000253825 215384720             0000
11625 2163307190034547870876 0 33697000264911-615501310             0000
11625 2163307190231367160905 0 26733000261462 456696400             0000
11625 2163307191549094666492 0 22313000102770-261753030             0000
ENDOFJOB
```

FIGURE 9. WEIGHTED DIFFERENTIAL CORRECTION, TEST CASE 1

SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL CORRECTION PRG   OCTOBER 23, 1963   PAGE 12

SATELLITE NO. 114 SATELLITE NAME : 61OMICRON ELEMENT SET NO. 20   TIME OF EPOCH 283.1576585 YEAR 1963

STA NO.	Y	MO	DY	TIME HH	MM	SS.SS	R J	RANGE RES, KM.	RT ASCEN RES, KM.	DECL. RES, KM.	AZIMUTH RES, KM.	ELEV. RES, KM.	RR RES. KM./SEC.	VECTOR MAG, KM.	DELTA T MIN.	U DEG.	BETA DEG.
214	[illegible]	7	1[illegible]	15	49	9.[illegible]	*	-.1214+2*			.2137+1	.644036	-.8986-2	.123+2	-.0121	41	.008
214	[illegible]	7	1[illegible]	2	51	[illegible].70		-.3740+1			-.1831+1	.4944+1	.9046-2	.646+1	.0257	157	.042
214	[illegible]	7	1[illegible]	0	54	24.77		-.[illegible]			-.1907+1	-.3391+1	-.2329-1	.913+1	-.0111	111	-.024
216	[illegible]	7	1[illegible]	22	[illegible]	[illegible]		-.[illegible]			-.1144+2	.1410+1	-.4540-2	.115+2	.0251	126	.022
214	[illegible]	7	1[illegible]	21	3	21.91		-.7692+1			-.1537+2	-.1280+1	-.1123-1	.172+2	.0350	98	-.043
214	[illegible]	7	1[illegible]	15	34	37.[illegible]	*	-.1287+2*			-.3057+1	.844-70	.5062-2	.132+2	-.0142	39	.031
214	[illegible]	7	1[illegible]	15	31	14.[illegible]		-.6560+1			.1373+1	-.1444+1	-.5106-2	.634+1	.0042	39	.032
330	[illegible]	7	1[illegible]	4	3	[illegible].55	*	-.3792+1			.1317+2	.2874+2*	-.5226-1*	.310+2	.0558	161	.006
330	[illegible]	7	1[illegible]	4	[illegible]	37.99		.9582+1			.4212+1	-.2602+1		.110+2	.0133	151	-.057
216	[illegible]	7	1[illegible]	2	9	[illegible].60		-.5266+1			-.294783	.3257+1	-.1279-1	.706+1	-.0134	127	.028
214	[illegible]	7	1[illegible]	[illegible]	22	29.47		-.4191+1			-.1370+1	-.3641+1	-.1111-1	.565+1	-.0316	115	-.013

SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL CORRECTION PRG   OCTOBER 23, 1963   PAGE 13

SATELLITE NO. 114 SATELLITE NAME: 61OMICRON ELEMENT SET NO. 20   TIME OF EPOCH 283.1576585 YEAR 1963

IF CONVERGED = THE NEXT CORRECTIONS WOULD BE....

OLD RMS KM.	NEW RMS KM.	OLD RMS2 KM/SEC	NEW RMS2 KM/SEC	DELTA N/N	DELTA AXN	DELTA AYN	DELTA UO RADIANS	DELTA NODE RADIANS	DELTA I RADIANS	DELTA A/M M**2/KGS
.493+3	.505+1	.499-4	.115-1	-.64649108-6	-.39349-4	.25344-5	-.74906-4	-.38137-4	.42452-4	

NO. OF RESIDUALS USED = 59 NO. OF RESIDUALS REJECTED = 4

	L DEGREES	TO DAYS IN YR	A EARTH RADII	E	I DEG.	NODE DEG.	OMEGA DEG.	MASS KGS.	PER ALT ST. MI.	PA MINUTES
OLD ELEMENTS WITH RESPECT TO OLD EPOCH	310.01175	280.67902	1.1469600	.0707	66.844	76.707	132.861	79.37300	550.2	103.818
OLD ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH	399.75377	283.13745	1.1462051	.0706	66.836	72.749	128.448	79.37300	543.7	103.715
NEW ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH	347.47129	283.13745	1.1455795	.0946	66.761	71.001	132.379	79.37300	534.9	103.664
NEW ELEMENTS WITH RESPECT TO NEW EPOCH	49.27857	282.68949	1.1481723	.0807	66.809	71.723	134.426	79.37000	550.3	103.977

FIGURE 10. OUTPUT, TEST CASE 1

```
                JOB     TEST CASE 2
                REM     SAT 116 4UHR OBS DC AND TIME PRED
SPSJOB  OBSWGTE 50
SPWDC                                                                         P
  TEST CASE 2 SAT 116                     1.1283    79.378                    2P
 -1.0      11 1200.7         111111 921   500.01000.   .5      1.5     2      3P
 1.0       11  7111                                                           6P
200.7        1.0    10                                                        7P
11                                                                           10P
116 20 161OMICRON1           10000     .007076397  66.844600456               E
116 20 21963          200.7000000000               310.611758                 E
116 20 3   .0720959506 78.707605     132.86108076    1.138844321              E
116 20 4                -2.419242667   -.703494501                            E
116 20 5 .0000                                                                E
116 20 6                                                                      E
116 20 7                            31202000000.00                            E
11622003903071804001800045000025490000118898
11625 039030718040308546 28270 19636000161120 457734220           0000
11625 216330718002219483 16360 35189000212596-519002660           0000
11625 216330718020924620 17640 30407000205147-491444840           0000
11625 216330718135114861 07000 09008000292007 080338390           0000
11625 216330718153457092 56640 15049000110276-208119980           0000
11625 216330718210321013 02460 01107000324938 022869110           0000
11625 216330718225513896 10190 05846000253825 215384720           0000
11625 216330719003454787 08760 33697000264911-615501390           0000
11625 216330719023136716 09050 20733000261462 456696400           0000
11625 216330719154909466 64920 22313000102770-261753030           0000
ENDOFJOB
```

FIGURE 11. WEIGHTED DIFFERENTIAL CORRECTION AND PREDICTION BY TIME, TEST CASE 2

SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL CORRECTION PRG OCTOBER 23, 1963 PAGE 10

SATELLITE NO. 114 SATELLITE NAME: 61OMICRON. ELEMENT SET NO. 21 TIME OF EPOCH 240.6999998 YEAR 1963

TIME (DAYS SINCE BEGINNING OF YEAR)

	X RADII	Y RADII	Z RADII	XDOT RAD/KEMIN	YDOT RAD/KEMIN	ZDOT RAD/KEMIN
	A RADII	E	I DEG	NODE DEG	OMEGA DEG	U DEG
	PHT DEG	LAMBDA DEG	H KM			
	SIGMA CT KM	SIGMA TT KM	SIGMA OP KM	SIGMA CT M/SEC	SIGMA TT M/SEC	SIGMA OP M/S
	SIGMA X KM	SIGMA Y KM	SIGMA Z KM	SIGMA XD M/SEC	SIGMA YD M/SEC	SIGMA ZD M/S
240.7760409						
	.485022237550	.2503187164-1	-.104449.643+1	.135458179051	.904651512450	.140432302374
	.114618248 5+1	.1053473716-1	.663943A3 46+2	.774379 37 7+2	.128113855 0+3	.279244364 9+3
	-.4449405648+2	.1800001832+3	.101746493+4			
	.2025297+1	.1225836+2	.3060104+1	.0737781+1	.203457 4+1	.343418 1+1
	.3114927+1	.1196269+2	.3360479+1	.4958913+1	.2624344+1	.892054 4+1

SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL CORRECTION PRG OCTOBER 23, 1963 PAGE 7

SATELLITE NO. 114 SATELLITE NAME: 61OMICRON. ELEMENT SET NO. 20 TIME OF EPOCH 240.6999998 YEAR 1963

DC CONVERGED • THE NEXT CORRECTIONS WOULD BE ...

OLD RMS KM.	NEW RMS KM.	OLD RMS2 KM/SEC	NEW RMS2 KM/SEC	DELTA N/N	DELTA AXN	DELTA AYN	DELTA UO RADIANS	DELTA NODE RADIANS	DELTA I RADIANS	DELTA A/M M**2/KGS
.491+3	.310+3	.497+1	.19273	.4845629-3	-.20825-2	-.39914-2	-.27649-1	-.10016-1	.72561-2	

NO. OF RESIDUALS USED = 33 NO. OF RESIDUALS REJECTED = 14

L DEGREES	TO DAYS IN YR	A EARTH RADII	E	I DEG.	NODE DEG.	OMEGA DEG.	MASS KGS.	PER ALT ST. MI.	P MINUTES
OLD ELEMENTS WITH RESPECT TO OLD EPOCH									
310.1175	240.69999	1.1469606	.00707	66.844	78.707	132.841	79.37800	550.2	103.818
OLD ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH									
310.01268	240.69999	1.1469605	.00707	66.844	78.707	132.840	79.37800	550.2	103.818
NEW ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH									
322.75603	240.69999	1.1463996	.00980	66.400	77.667	125.174	79.37800	535.6	103.740
NEW ELEMENTS WITH RESPECT TO NEW EPOCH									
309.74603	240.69969	1.1463996	.00980	66.400	77.667	125.174	79.37800	535.6	103.740

FIGURE 12. OUTPUT, TEST CASE 2

```
                JOB    .TEST CASE 3
                REM     SAT 116 40HR OBS DC AND STA PRED
SPSJOB  OBSWGTE 50
SPWDC                                                                       . 1P
  TEST CASE 3 SAT 116                     1.1283    79.378                    2P
 -1.0      11 2              111111 91         1000.   .5      1.5     2      3P
 2.0       11 15  02165.C       5.0       1.0       1.0                       8P
12                                                                           10P
116 20 1610MICRON1             10000   .0070763397  66.8446 00456              E
116 20 21963          200.7000000000               310.6117 58                 E
116 20 3   .0720959506 78.707605     132.86108076    1.1388 44321              E
116 20 4                -21.41924267    -.703494501                            E
116 20 5  .0000                                                                E
116 20 6                                                                       E
116 20 7                            31201000000.00                             E
11622003903071804001800045000025490000118898
11625 03903071804030854628270 19636000161120 457734220           0000
11625 21633071800221948316360 35189000212596-519002660           0000
11625 21633071802092462017640 30407000205147-491444840           0000
11625 21633071813511486107000 09008000292007 080338390           0000
11625 21633071815345709256640 15049000110276-208119980           0000
11625 21633071821032101302460 01107000324938 022869110           0000
11625 21633071822551389610190 05846000253825 215384720           0000
11625 21633071900345478708760 33697000264911-615501390           0000
11625 21633071902313671609050 20733000261462 456696400           0000
11625 21633071915490946664920 22313000102770-261753030           0000
ENDOFJOB
ENDSCHED
```

FIGURE 13. UNWEIGHTED DIFFERENTIAL CORRECTION AND PREDICTION BY STATION, TEST CASE 3

```
SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL CORRECTION PRG      OCTOBER 23, 1963                                          PAGE  21

SATELLITE NO. 114  SATELLITE NAME: 63OMICRON4  ELEMENT SET NO.  21                  TIME OF EPOCH  280.6591372  YEAR 1963

TIME (DAYS SINCE BEGINNING OF YEAR)
                      X    RADII         Y    RADII          Z    RADII        XDOT RAD/KEMIN  YDOT RAD/KEMIN  ZDOT RAD/KEMIN
                      A    RADII         E                   I    DEG          NODE     DEG    OMEGA    DEG    U      DEG
                      PHI  DEG           LAMBDA DEG          H    KM
                      RHO  KM            RH-DOT KM/SEC       AZIM DEG          ELEV     DEG

   280.9632683     -.305883298919     -.737121556093      .795840084648     .4458109510-1   -.759217457164   -.561543207615
                    .1146751360+1      .7576149881-2      .6660967207+2     .7612458040+2    .1327126474+3    .1305028047+3
                    .4456237138+2      .3070301043+3      .8911154198+3
                    .1836524016+4      .2860139591+1      .7553265846+2     .2203723900+2

   280.2650550     -.300136595051     -.491833068791      .752612993306     .8902453005-1   -.718738433207   -.600649233907
                    .1140700759+1      .7705153838+2      .6661124642+2     .7612469296+2    .1324797480+3    .1339694583+3
                    .4159968670+2      .3093557437+3      .8899897342+3
                    .2040138370+4      .7864800218+1      .8679244979+2     .1796077333+2

   280.9506541     -.492516551110     -.945953631958      .706560277845     .1129384205 92  -.682556135735   -.637511126115
                    .1147050970+1      .7835846327-2      .6681282878+2     .7612301471+2    .1324007635+3    .1374367134+3
                    .3859355936+2      .3114731887+3      .8890789601+3
                    .2294402713+4      .4567619563+1      .9540598219+2     .1347811129+2

   280.9573457     -.483052177875     -.793287390016      .657854200388     .1352661 2067    -.643813872241   -.671985984779
                    .1147199676+1      .7964923625-2      .6681439456+2     .7612154737+2    .1324666186+3    .1409045685+3
                    .3549142223+2      .3133896479+3      .8883974383 23
                    .2523757152+4      .5046767473+1      .1019990495+3     .0518056631+1

SPECIAL PERTURBATIONS WITH WEIGHTED DIFFERENTIAL CORRECTION PRG      OCTOBER 23, 1963                                          PAGE  13

SATELLITE NO. 114  SATELLITE NAME: 63OMICRON4  ELEMENT SET NO.  20                  TIME OF EPOCH  280.6591372  YEAR 1963

NO CONVERGED - THE NEXT CORRECTIONS WOULD BE ...

OLD RMS   NEW RMS   OLD RMS2  NEW RMS2     DELTA A/N     DELTA AXN   DELTA AYN   DELTA UO    DELTA NODE   DELTA I    DELTA A/M
  KM.       KM.      KM/SEC    KM/SEC                                            RADIANS     RADIANS      RADIANS    M**2/KGS

 .492+3    .460+1    .499-4    .132-1      .2812376R-6   .13032-4    .03543-4    .14589-3    -.30485-4    -.42580-4

NO. OF RESIDUALS USED =   41   NO. OF RESIDUALS REJECTED =    6

        L          TC            A              E         I        NODE      OMEGA       MASS       PER ALT    PA
     DEGREES    DAYS IN YR   EARTH RADII                DEG.      DEG.      DEG.        KGS.       ST. MI.    MINUTES

OLD ELEMENTS WITH RESPECT TO OLD EPOCH
     310.57175   280.65909    1.1469403    .02707    66.844    78.767    132.841    79.37800    550.2    103.818

OLD ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH
     106.77542   280.67943    1.1479493    .02754    66.855    78.815    135.676    79.37800    552.1    103.955

NEW ELEMENTS WITH RESPECT TO INTERMEDIATE EPOCH
     119.28307   280.65943    1.1471676    .02843    66.813    77.025    136.609    79.37800    544.6    103.838

NEW ELEMENTS WITH RESPECT TO NEW EPOCH
     119.28407   280.65943    1.1471672    .02843    66.313    77.195    135.419    79.37800    544.6    103.838
```

FIGURE 14. OUTPUT, TEST CASE 3

```
70WEIGHT 11-8-2
0390     13.200 .0220  .8896  .4014   11-8-2
2160      6.583 .02796 .28266 .117    11-8-2
ENDSIGMA 11-8-2
```

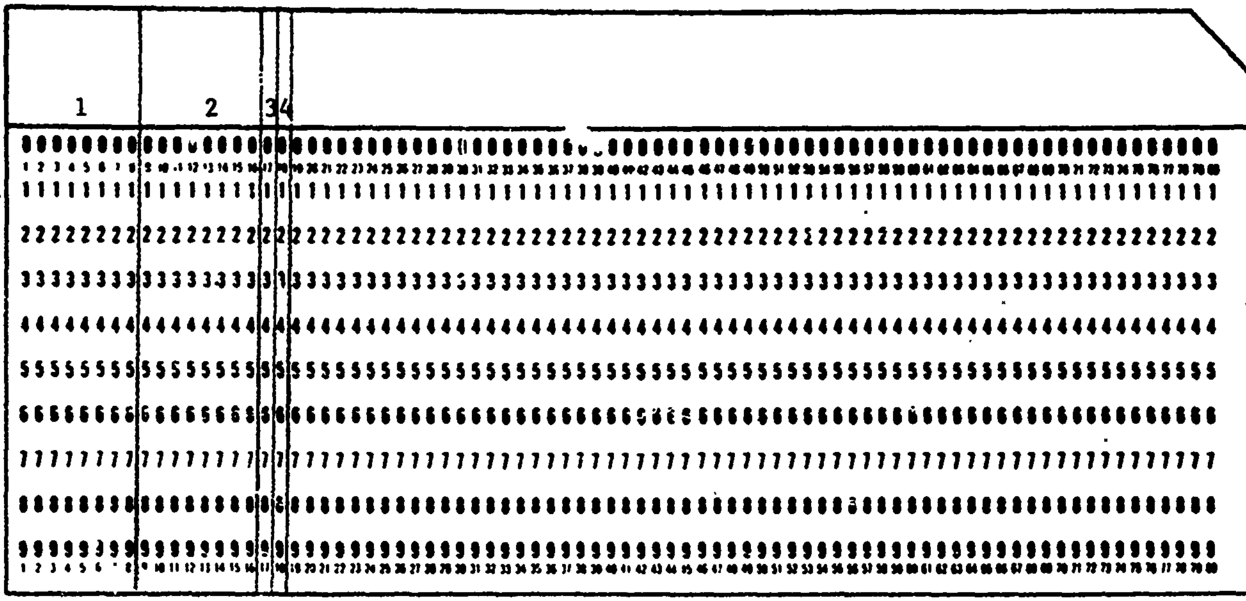
FIGURE 15. WEIGHT TAPE

APPENDIX I

PROGRAM PARAMETER CARDS

Code information pertinent to the parameter card formats is as follows:

Code	Contents
1	Alphanumeric information
2	Integers
3	Floating point numbers XX...X.XX...X (decimal anywhere in the field) ±.XXX...X+YY XXXXX.....X (no decimal)
4	Flag Δ or 0: not to be computed 1: to be computed



Field	Column	Contents
1	1 - 8	SPSJOB
2	9 - 16	OBSWGTE Δ
3	17	INPUT OPTIONS (See following page)
4	18	0 OUTPUT OPTION

FIGURE 16. SPSJOB CARD

SCHEDULE TAPE INPUT OPTIONS

(Reference SPSJØB Cards)

Option	Sensor Cards	Element Cards	Observation Cards	Parameter Cards	Satellite Cards
0	N	N	N*	Y	Y
1	N	Y	N*	Y	N
2	Y	N	N*	Y	Y
3	Y	Y	N*	Y	N
4	N	N	Y	Y	Y
5	N	Y	Y	Y	N
6	Y	N	Y	Y	Y
7	Y	Y	Y	Y	N
8	N	Y	N	Y	N
9	N	N	N	Y	Y

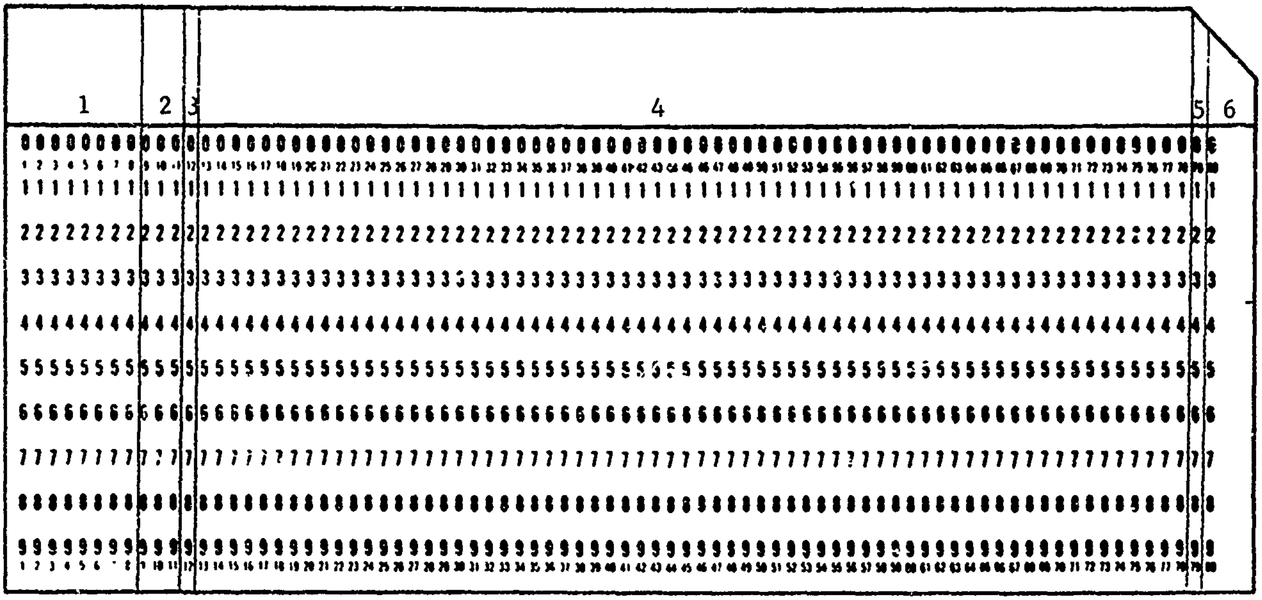
N = No

Y = Yes

Options 0-7 refer to differential correction and (optionally) prediction
8-9 refer to prediction only

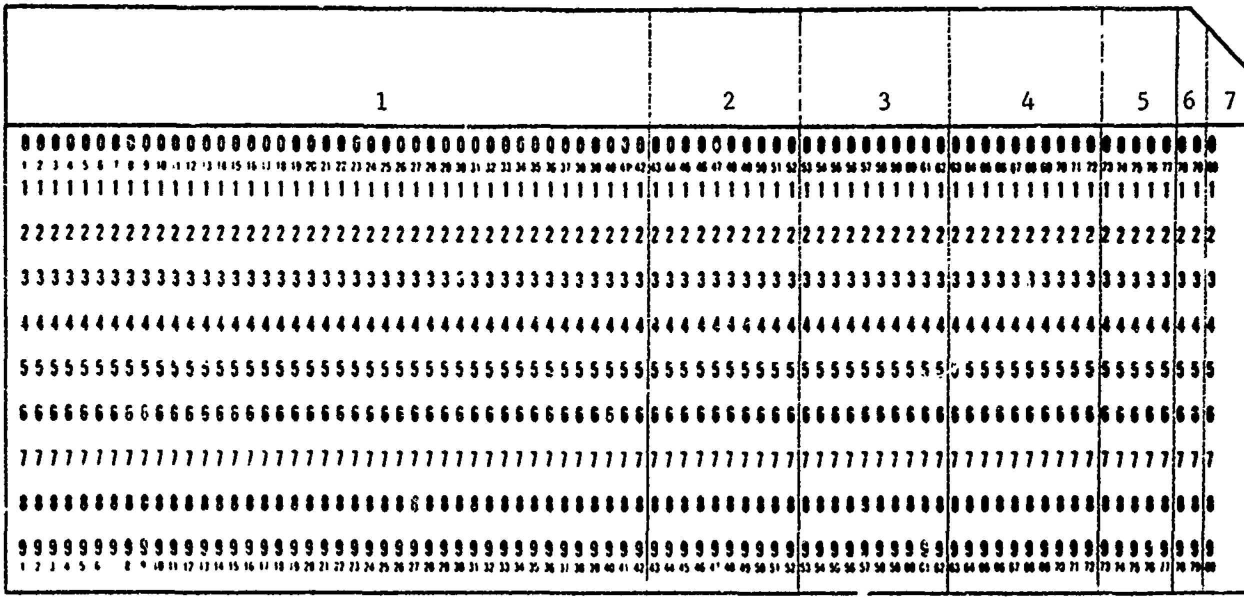
*Observations are from the SRADU tape.

NOTE: Whenever N appears, data are obtained from data files (except for satellite number cards). In the case of sensors and elements, the data source is the SPS B-2 SEAI tape. When elements are input from the SEAI tape, the Schedule Tape input must contain satellite number cards. If observation cards are not input from the schedule tape, they are obtained from the SRADU tape.



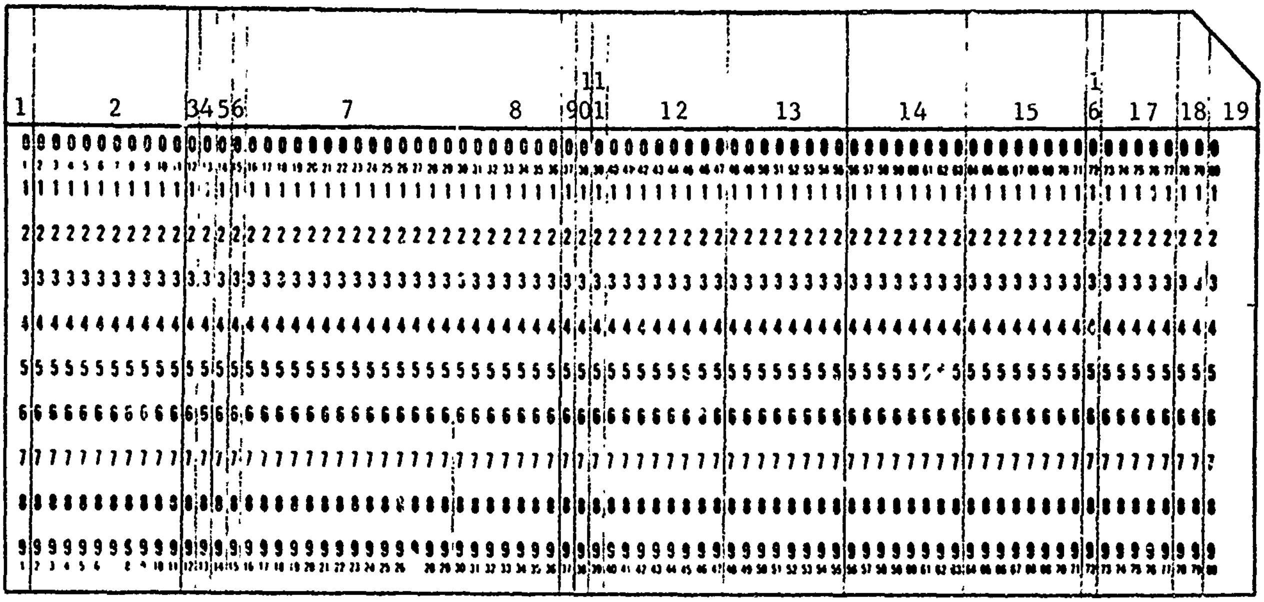
Field	Column	Contents
1	1 - 8	SPWDC ΔΔΔ
2	9 - 11	Satellite Number (optional)
3	12	0 or Δ - Weighting Tape Input 1 - Use Sensor Weighting Data in OBSWGT Weight File
4	13 - 78	Not used
5	79	0 or Δ - Use Observation Weights in SPWDC 1 - Do Not Use Observation Weights
6	80	P

FIGURE 1[illegible]. OBSWGT PARAMETER CARD



Field	Column	Contents	Code
1	1 - 42	Remarks	1
2	43 - 52	Diameter of Satellite (Meters)	3
3	53 - 62	Mass of Satellite (Kg)	3
4	63 - 72	γ (Reflectivity)	3
5	73 - 77	Not used	
6	78 - 79	Δ 2	
7	80	P	

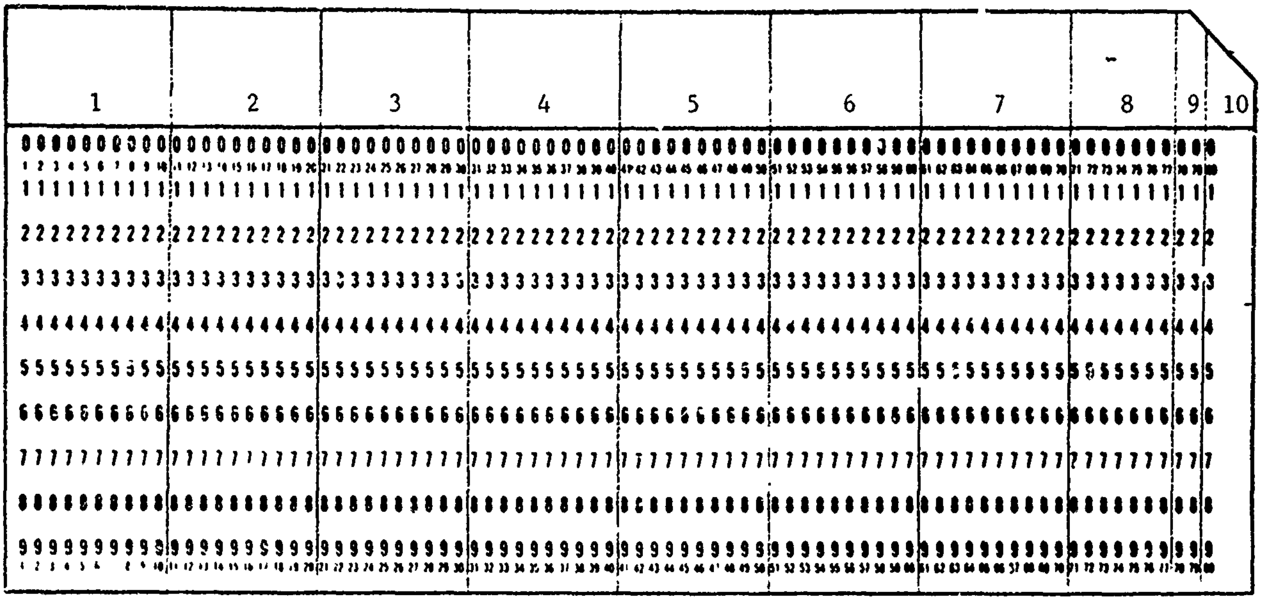
FIGURE 18. VEHICLE CHARACTERISTIC CARD



Field	Column	Contents	Code
1	1	Δt Integration Mode (Δ- variable, 1-fixed)	2
2	2 - 11	Δt Minutes (positive or negative)	3
3	12	Bulge Perturbation	4
4	13	Drag Perturbation	4
5	14	Radiation Pressure Perturbation	4
6	15	New Epoch Mode (Δ- Rev)(1-Time)(2-Time of Last Obs)	
7	16 - 29	t_p (time in days since beginning of year)(Absolute Revolution Number)	3
8	30 - 36	Elements to Correct: n, a_{xN}, a_{yN}, U_o, Ω, i, m	4
9	37	Max. Number of Corrections	2
10	38	n Only Correction on the First Pass*	4
11	39	Δq Check*	4
12	40 - 47	Max. Δ q (KM)	3
13	48 - 55	ABSMX (KM)	3
14	56 - 63	ABSMX2 (KM/Sec)	3
15	64 - 71	n (RMS Multiplier)	3
16	72	Residual Output (Δ -Never)(1 - First/Last Set)(2-Every Set)	
17	73 - 77	Not Used	
18	78 - 79	Δ 3	
19	80	P	

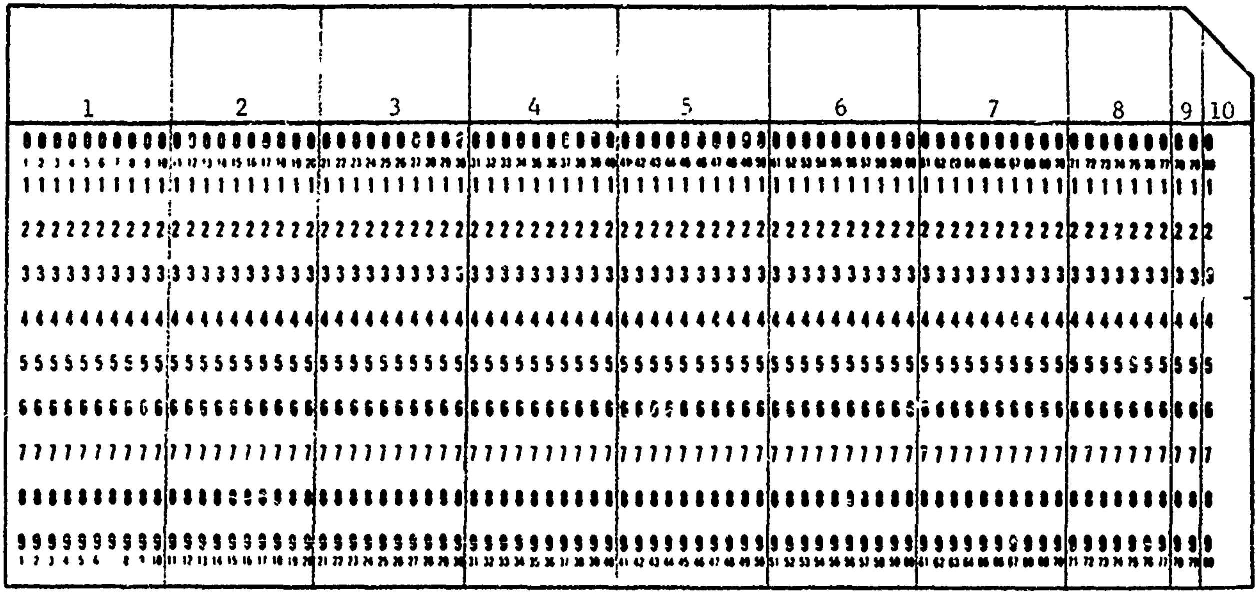
*Applies only if 6-7 Elements are to be corrected.

FIGURE 19. DIFFERENTIAL CORRECTION CONTROL CARD



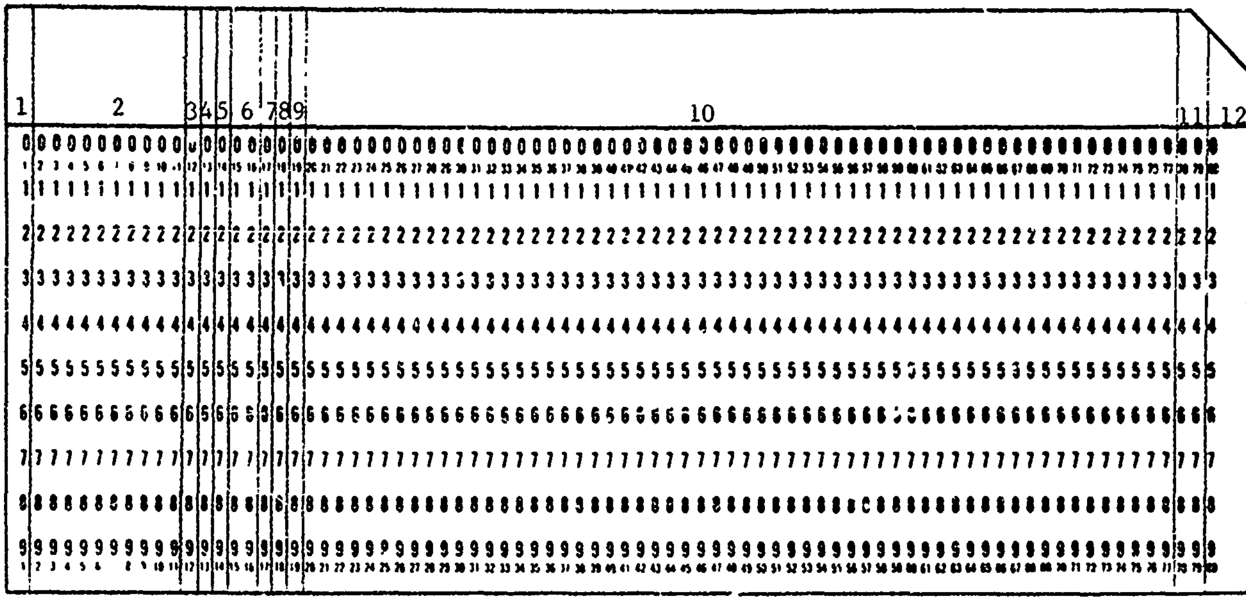
Field	Column	Contents	Code
1	1 - 10	a_L (Absolute Error Criteria)	3
2	11 - 20	a_{a_x}	3
3	21 - 30	a_{a_y}	3
4	31 - 40	a_{a_z}	3
5	41 - 50	a_{h_x}	3
6	51 - 60	a_{h_y}	3
7	61 - 70	a_{h_z}	3
8	71 - 77	Not Used	
9	78 - 79	Δ 4	
10	80	P	

FIGURE 2[illegible]. ABSOLUTE ERROR CONTROL CARD



Field	Column	Contents	Code
1	1 - 10	r_L (Relative Error Criteria)	3
2	11 - 20	r_{a_x}	3
3	21 - 30	r_{a_y}	3
4	31 - 40	r_{a_z}	3
5	41 - 50	r_{h_x}	3
6	51 - 60	r_{h_y}	3
7	61 - 70	r_{h_x}	3
8	71 - 77	Not Used	
9	78 - 79	$\Delta 5$	
10	80	P	

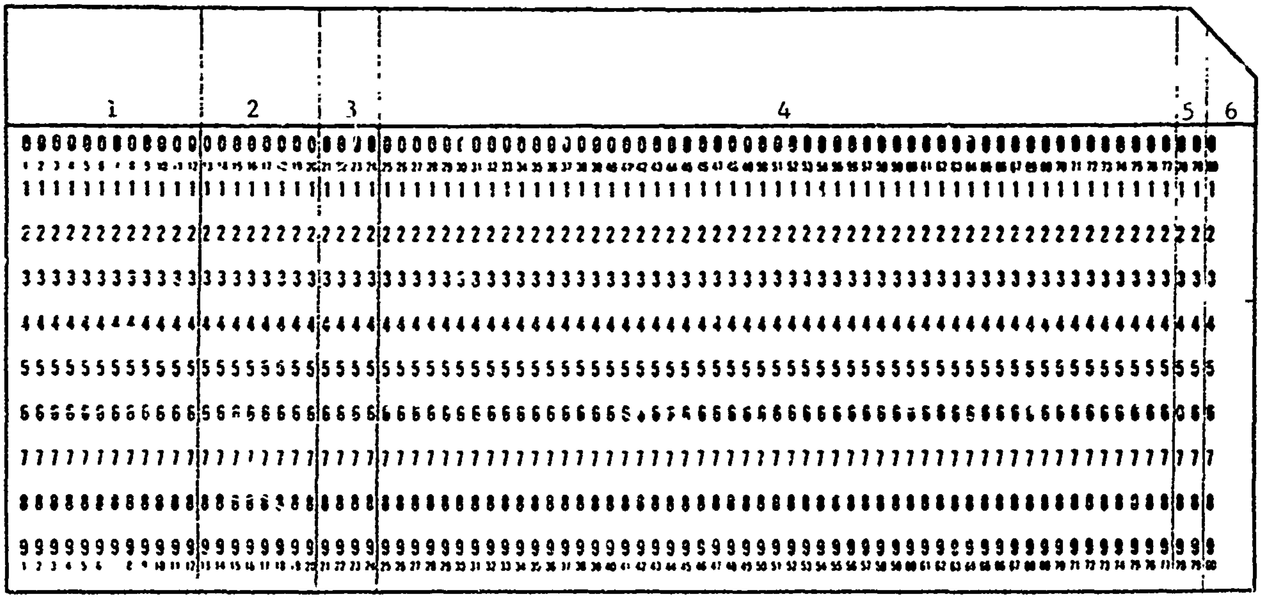
FIGURE 21. RELATIVE ERROR CONTROL CARD



Field	Column	Contents	Code
1	1	Δt Integration Mode (Δ- variable) (1 - fixed)	2
2	2 - 11	Δt (Min)(may be negative if desired)	3
3	12	Bulge Perturbation	4
4	13	Drag Perturbation	4
5	14	Radiation Pressure Perturbation	4
6	15 - 16	Print Flag $\left\{\begin{array}{l} 1\text{-}t,\ \underline{r},\ \dot{\underline{r}} \\ 2\text{-}t, a, e, i, \Omega, \omega, U, \\ 4\text{-}t, \Phi,\ \lambda_E,\ h \end{array}\right\}$	Add flags for combination of data
7	17	Binary Tape Output	4
8	18	Prediction Reliability*	4
9	19	Punched Cards ($\underline{r}$,t)	4
10	20 - 77	Not Used	
11	78 - 79	Δ6	
12	80	P	

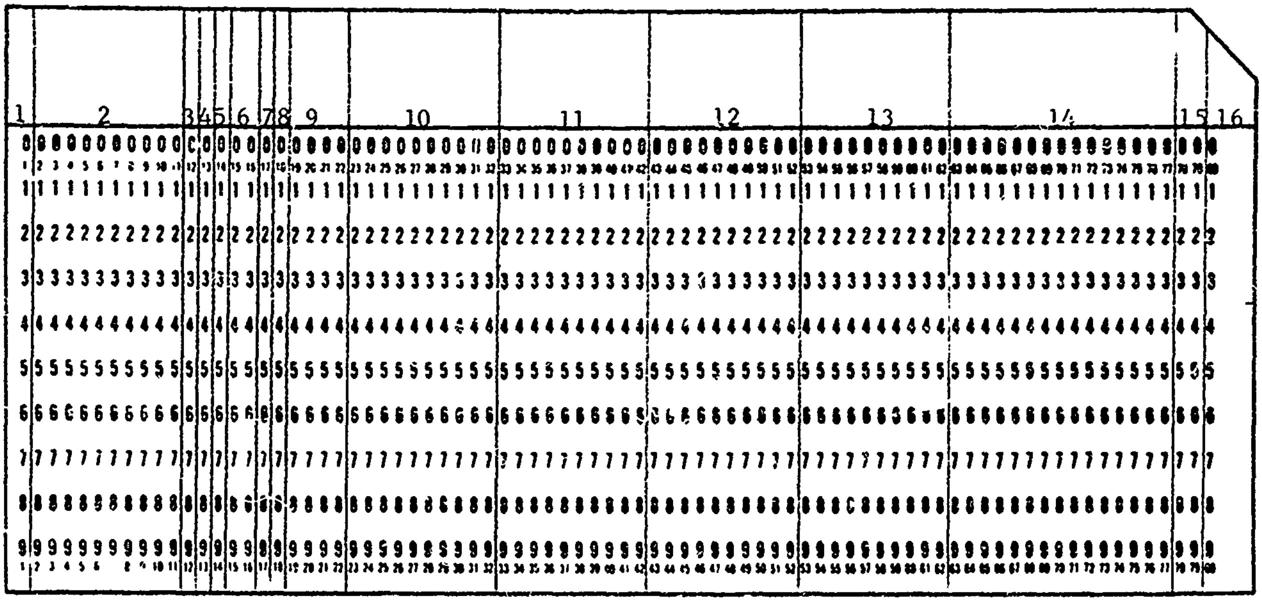
*Must be a weighted 6 element or 7 element with drag differential correction run.

FIGURE 22. TIME PREDICTION CARD #1



Field	Column	Contents	Code
1	1 - 12	Time in days since beginning of year	3
2	13 - 20	Δt (Min)	3
3	21 - 24	Number of output points	3
4	25 - 77	Not Used	
5	78 - 79	$\Delta 7$	
6	80	P	

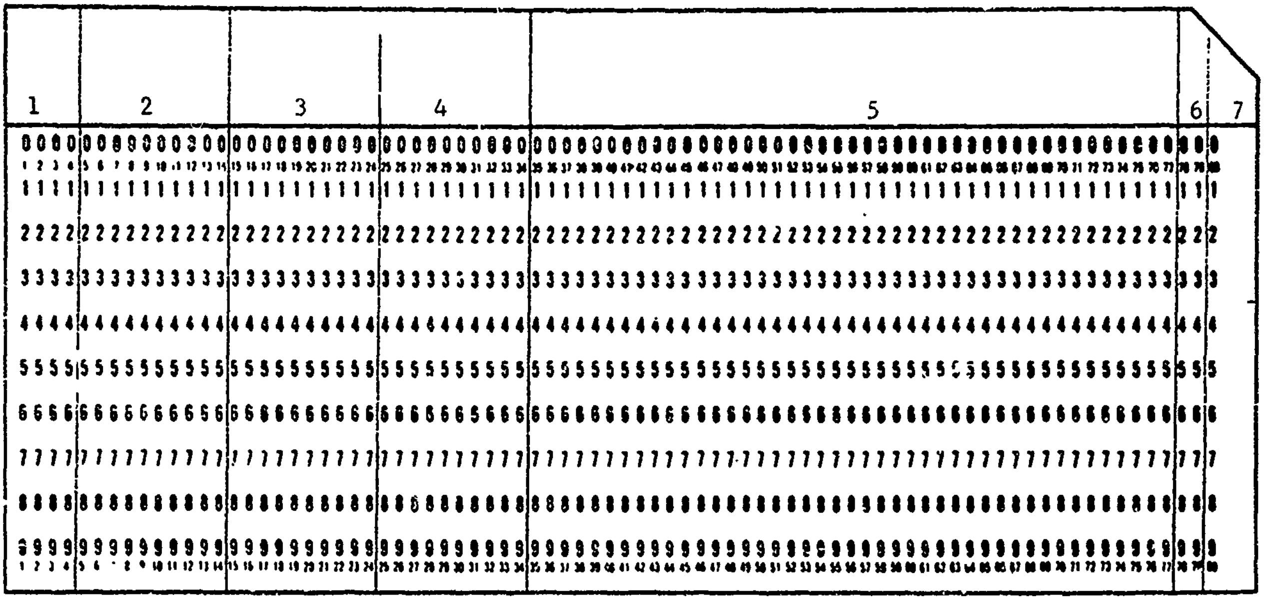
FIGURE 2[illegible]. TIME PREDICTION CARD #2



Field	Column	Contents	Code
1	1	Δt Integration Mode (Δ- variable) (1- fixed)	2
2	2 - 11	Δt (Min)	3
3	12	Bulge Perturbation	4
4	13	Drag Perturbation	4
5	14	Radiation Pressure Perturbation	4
6	15 - 16	Print Flag 1-t, $\underline{r}$, $\dot{\underline{r}}$; 2-t, a, e, i, Ω, ω, U; 4-t, Φ, λ_E, h; 8-t, ρ, $\dot{\rho}$, A, h	Add flags for combination of data
7	17	Binary Tape Output	4
8	18	Prediction Reliance*	4
9	19 - 22	Sensor Number ($\Delta\Delta\Delta\Delta$ - Station Pred. Card 2 needed) (XXXX - SEAI Tape)	1
10	23 - 32	K - No. of Passes	2
11	33 - 42	ΔT (Min) (Either Side of Closest Approach)	3
12	43 - 52	Δt* (Min) (Output per Δt*)	3
13	53 - 62	Minimum h (Deg)	3
14	63 - 77	Not Used	
15	78 - 79	Δ8	
16	80	P	

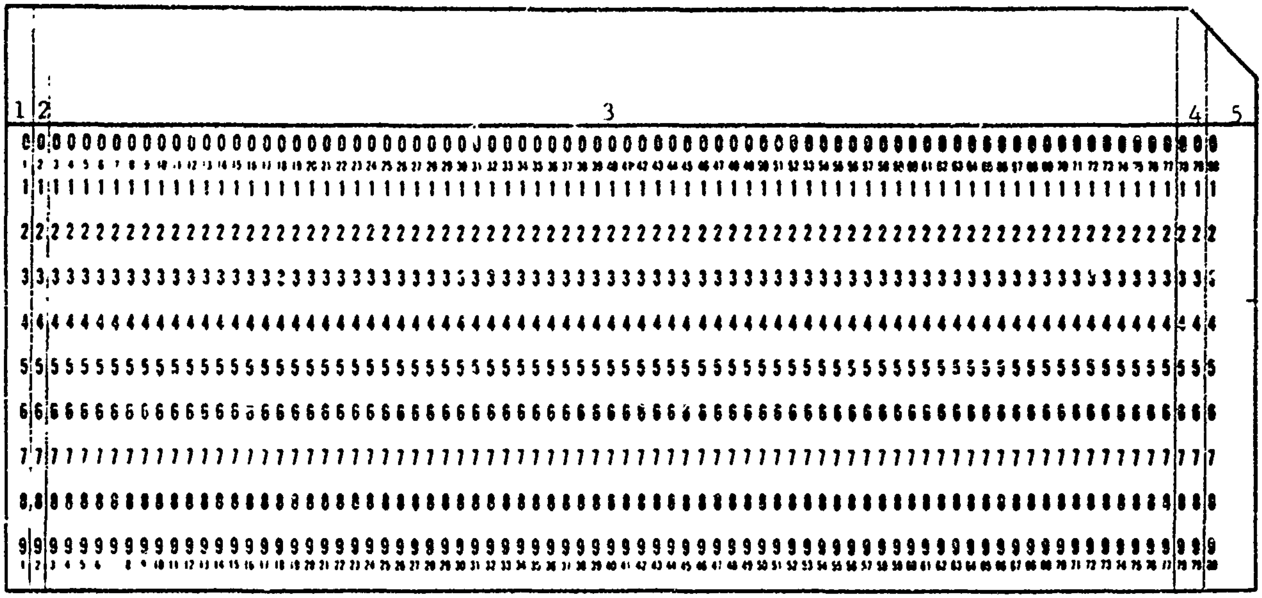
*Must be a weighted 6 element or 7 element with drag differential correction run.

FIGURE 24. STATION PASS PREDICTION CARD #1



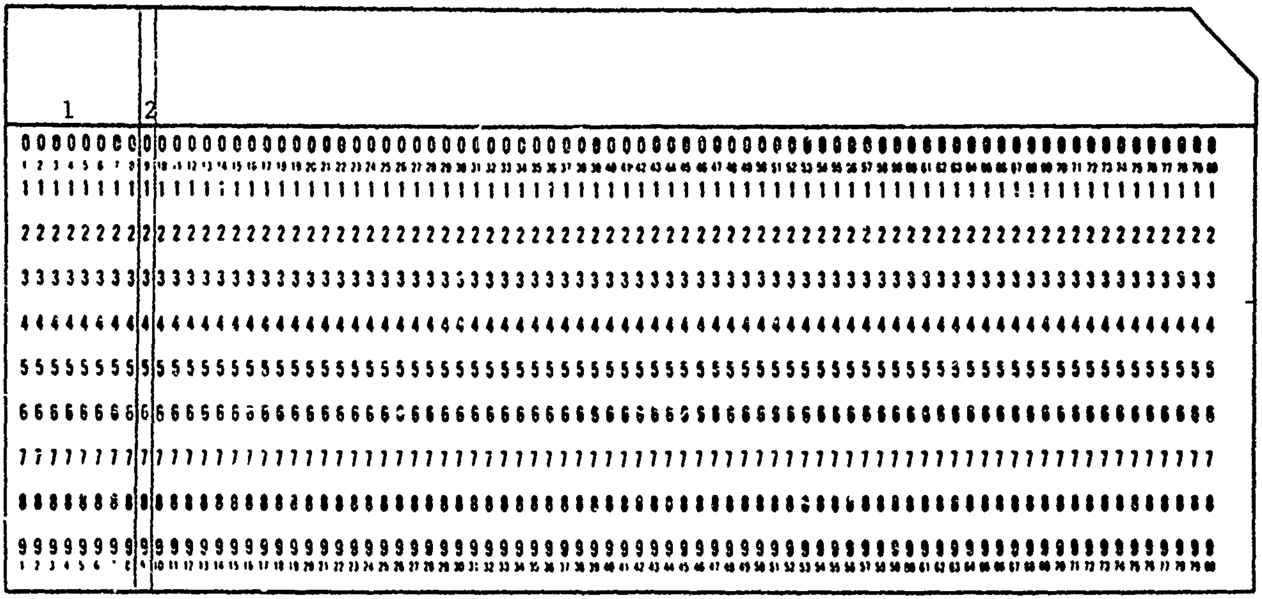
Field	Column	Contents	Code
1	1 - 4	Station No.	1
2	5 - 14	Φ_N Latitude (Degrees)	3
3	15 - 24	λ_ω Longitude West (Degrees)	3
4	25 - 34	H (Meters)	3
5	35 - 77	Not Used	
6	78 - 79	$\Delta 9$	
7	80	P	

FIGURE 25. STATION PASS PREDICTION CARD #2



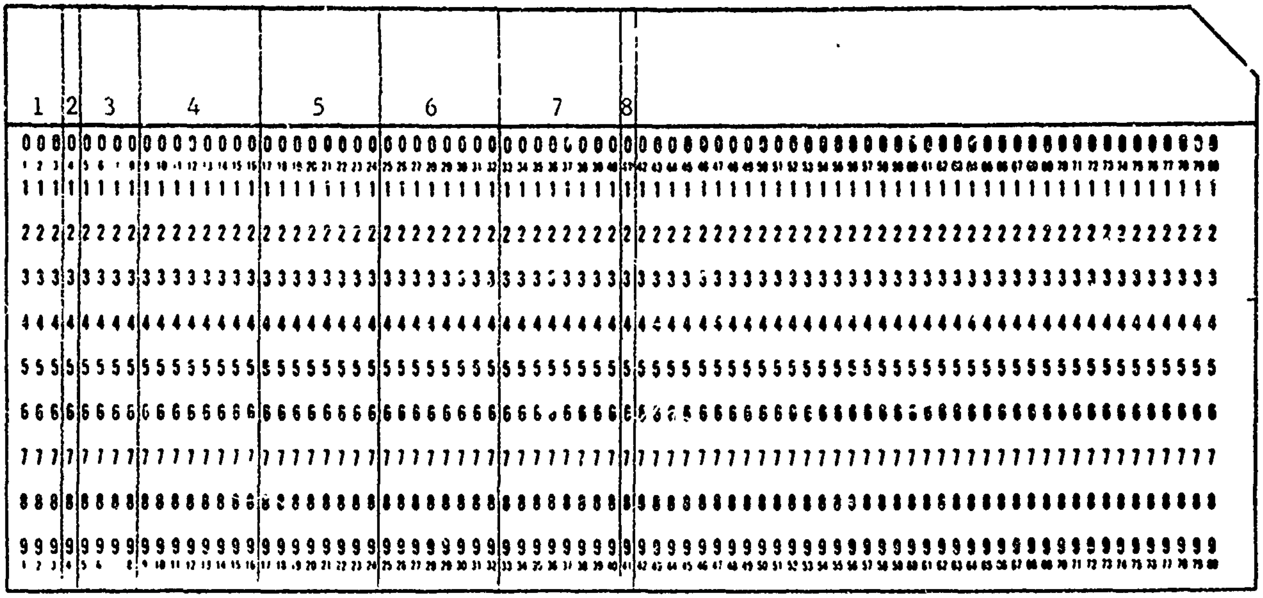
Field	Column	Contents	Code
1	1	DC Flag	4
2	2	Prediction ($\overset{\Delta}{0}$ - no)(1-Time)(2-Station)	2
3	3 - 77	Not Used	
4	78 - 79	10	
5	80	P	

FIGURE 26. PROGRAM EXECUTION CARD



Field	Column	Contents
1	1 - 8	If ID CARD - 70WEIGHT If END CONTROL CARD - ENDSIGMA
2	9	11_{8_2} Multiple Punch

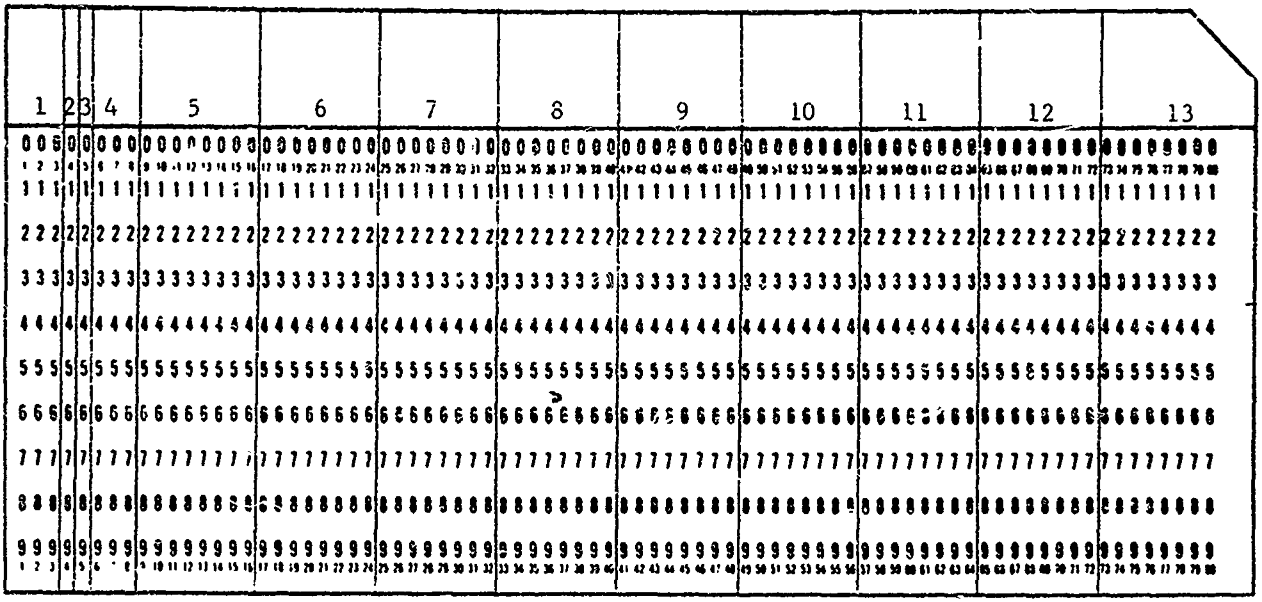
FIGURE 2 . WEIGHT TAPE CONTROL CARD



Field	Column	Contents
1	1 - 3	Sensor Number
2	4	0 - Sigma Data is for all observations from a specified sensor 2 - Sigma Data is for the next observation only
3	5 - 8	Not Used
4	9 - 16	σ_1 ρ(km)
5	17 - 24	σ_2 $\dot{\rho}$(km/sec)
6	25 - 32	σ_3 A or α (degrees)
7	33 - 40	σ_4 h or δ (degrees)
8	41	11_{8_2} Multiple Punch

NOTE: If a sigma is not input, leave the field blank

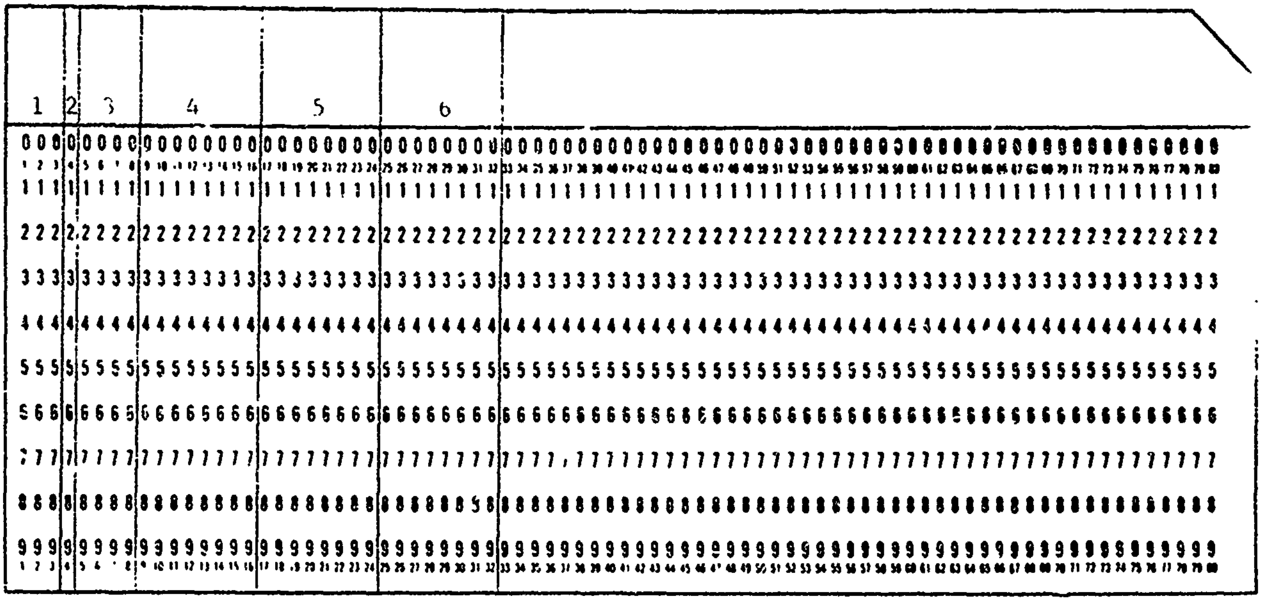
FIGURE 23 OBSERVATION OR SENSOR WEIGHTING CARD



Field	Column	Contents
1	1 - 3	Sensor Number
2	4	1 - Parameter Data is for a function
3	5	Δ or 0 - if the last parameter is contained on this card*
		1 - if parameters follow on next card
4	6 - 8	Not Used
5	9 - 16	Function Identifier (left adjusted)
6 - 13	17 - 80	P_1 through P_8*

*The last parameter must be followed by the 11_{8_2} multiple punch in the next field or on the next card if necessary.

FIGURE 29. FUNCTION WEIGHTING CARD 1



Field	Column	Contents
1	1 - 3	Sensor Number
2	4	1 - Parameter data is for a function
3	5 - 8	Not Used
4	9 - 16	Function Identifier
5	17 - 24	P_9
6	25 - 32	P_{10} *

*The last parameter must be followed by the 11_{8_2} multiple punch in the next field.

FIGURE 30. FUNCTION WEIGHTING CARD 2

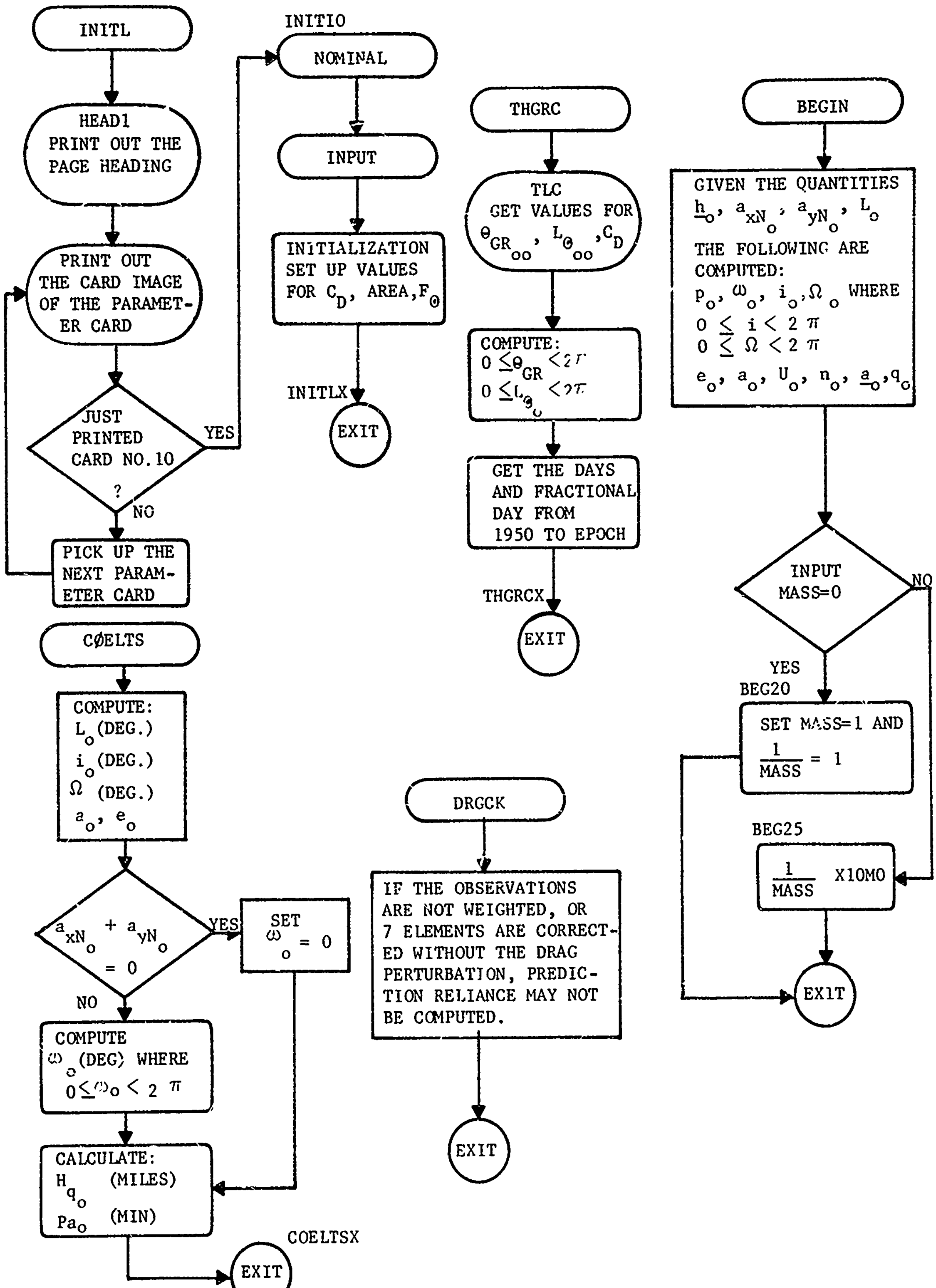


FIGURE 31. DETAILED FLOW DIAGRAMS

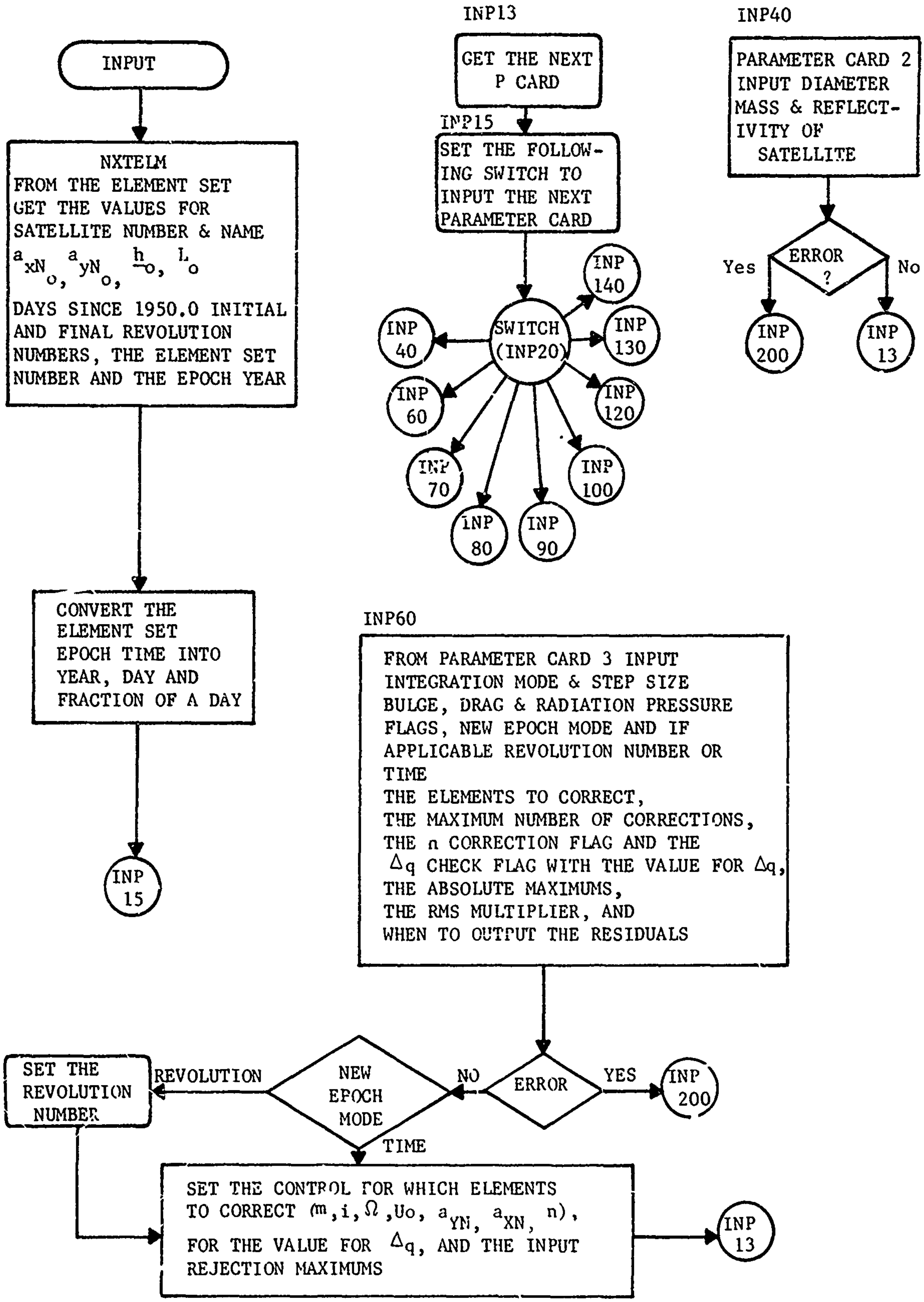


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

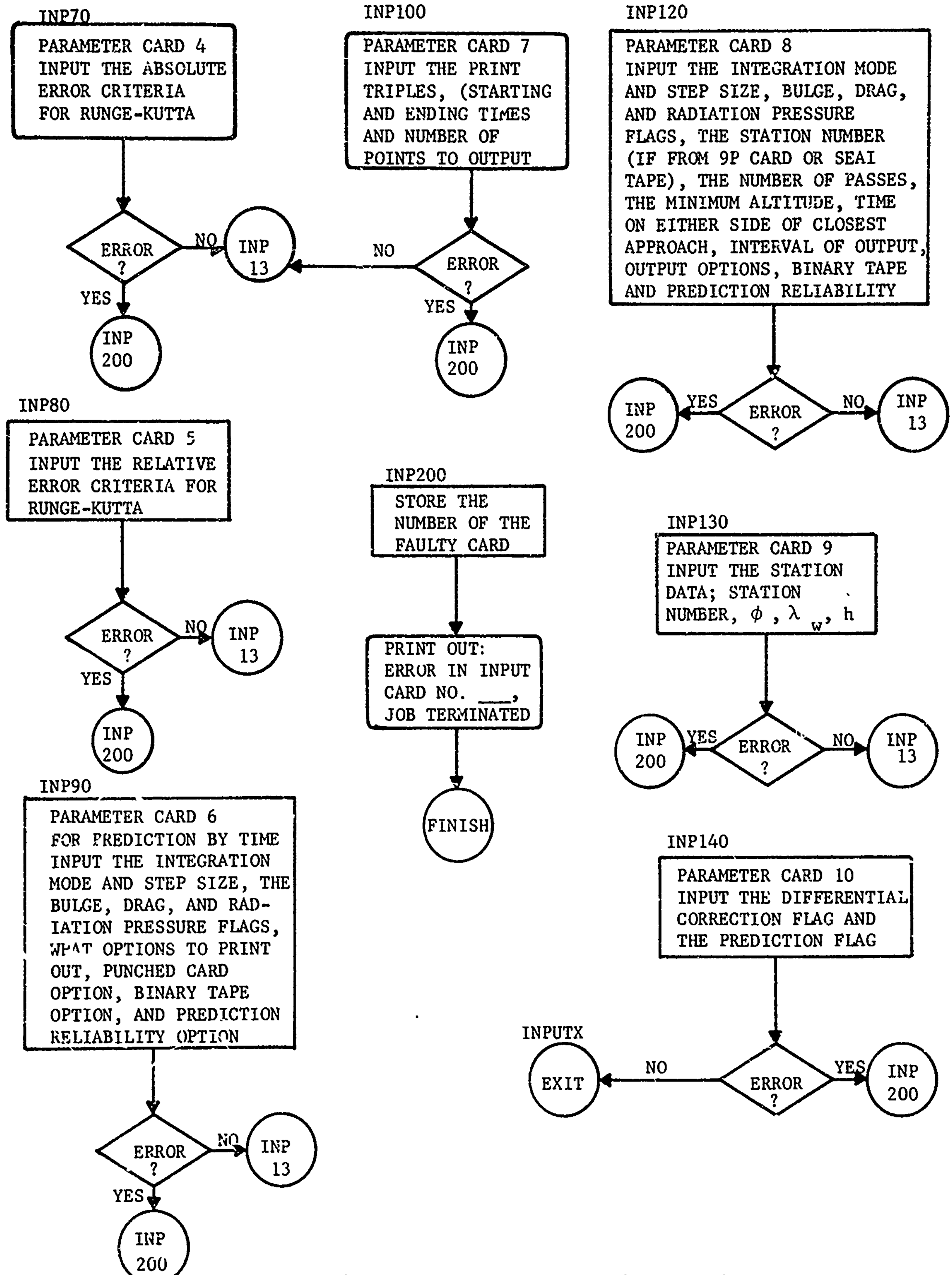


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

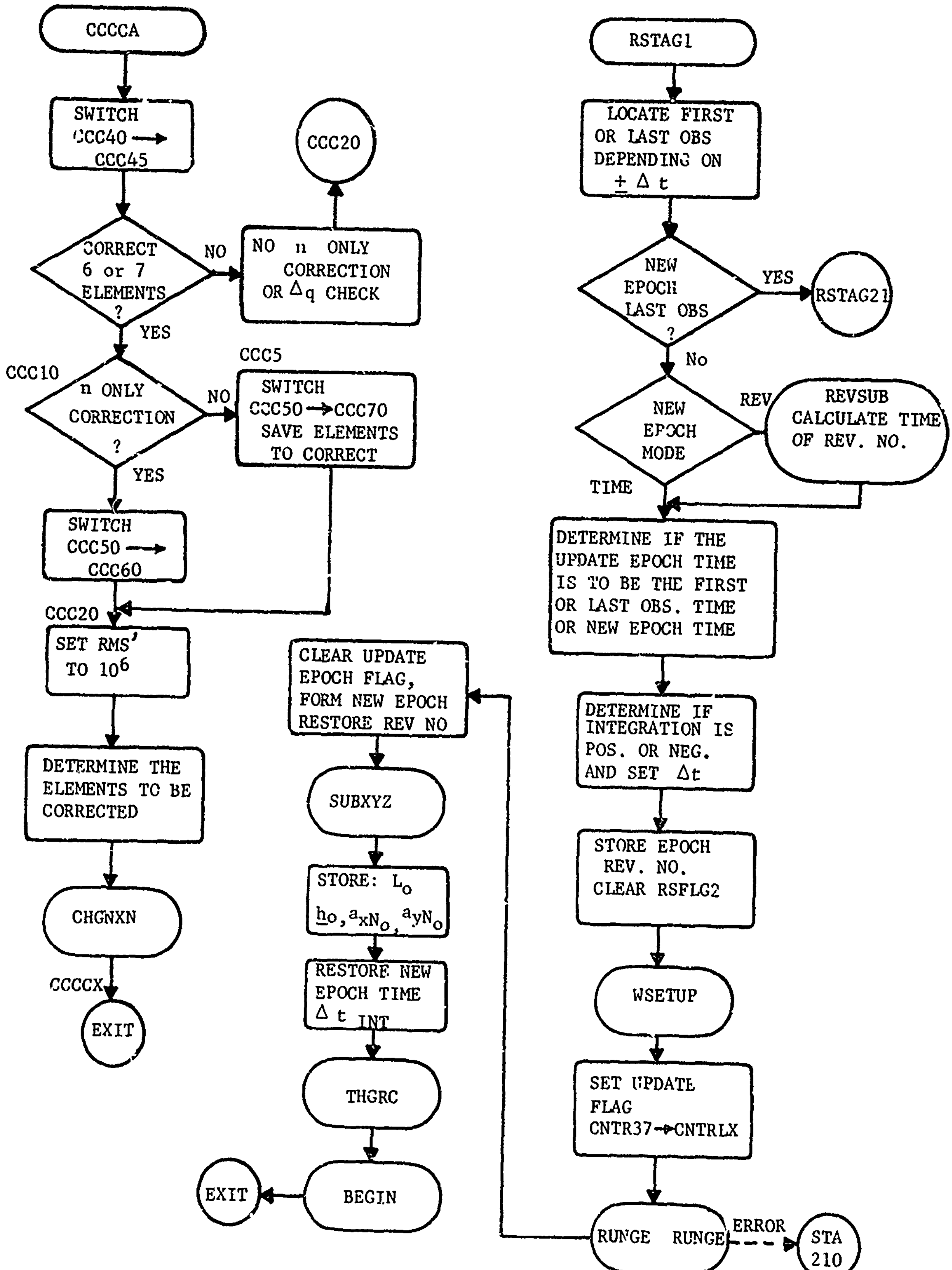


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

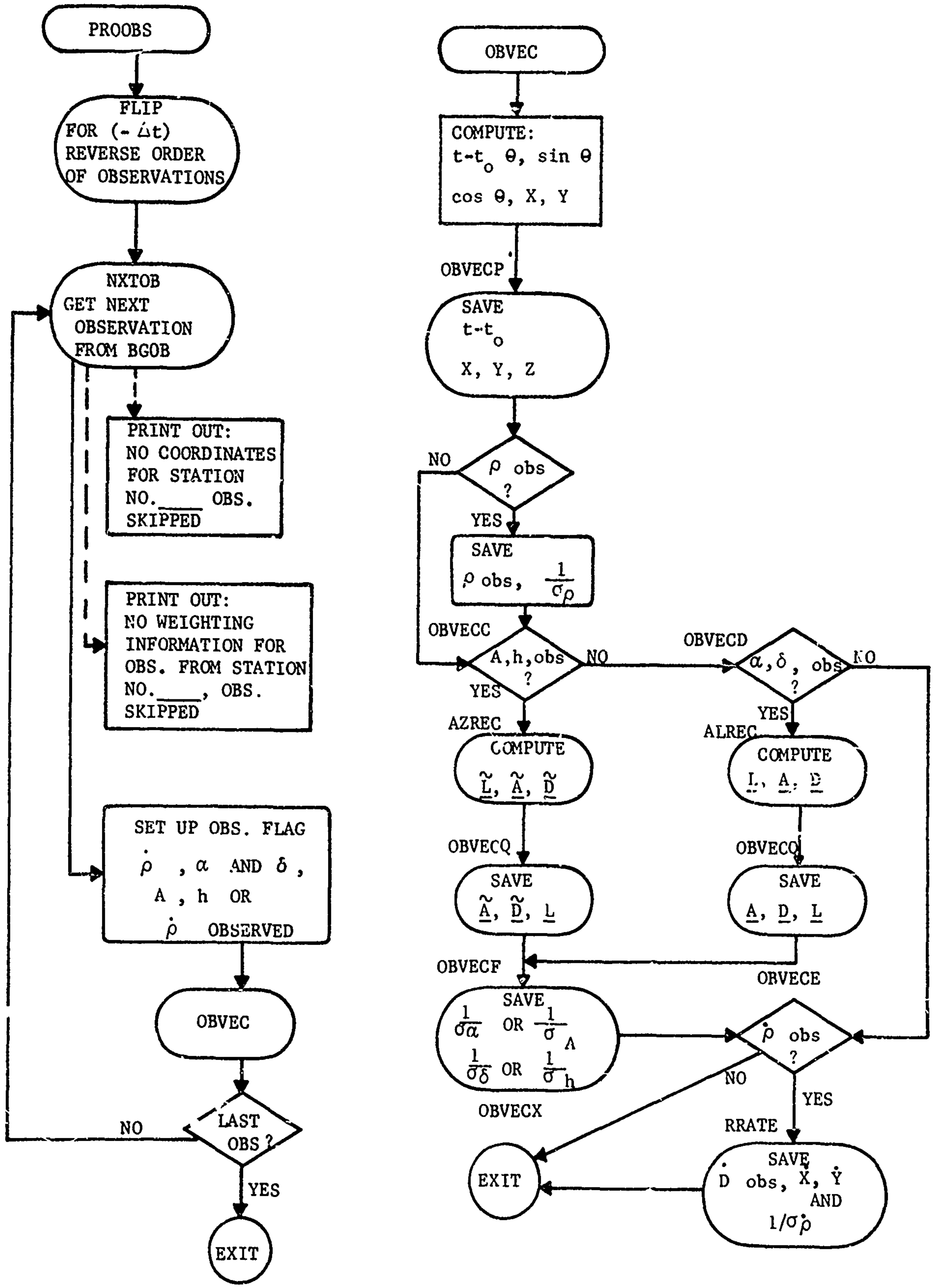


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

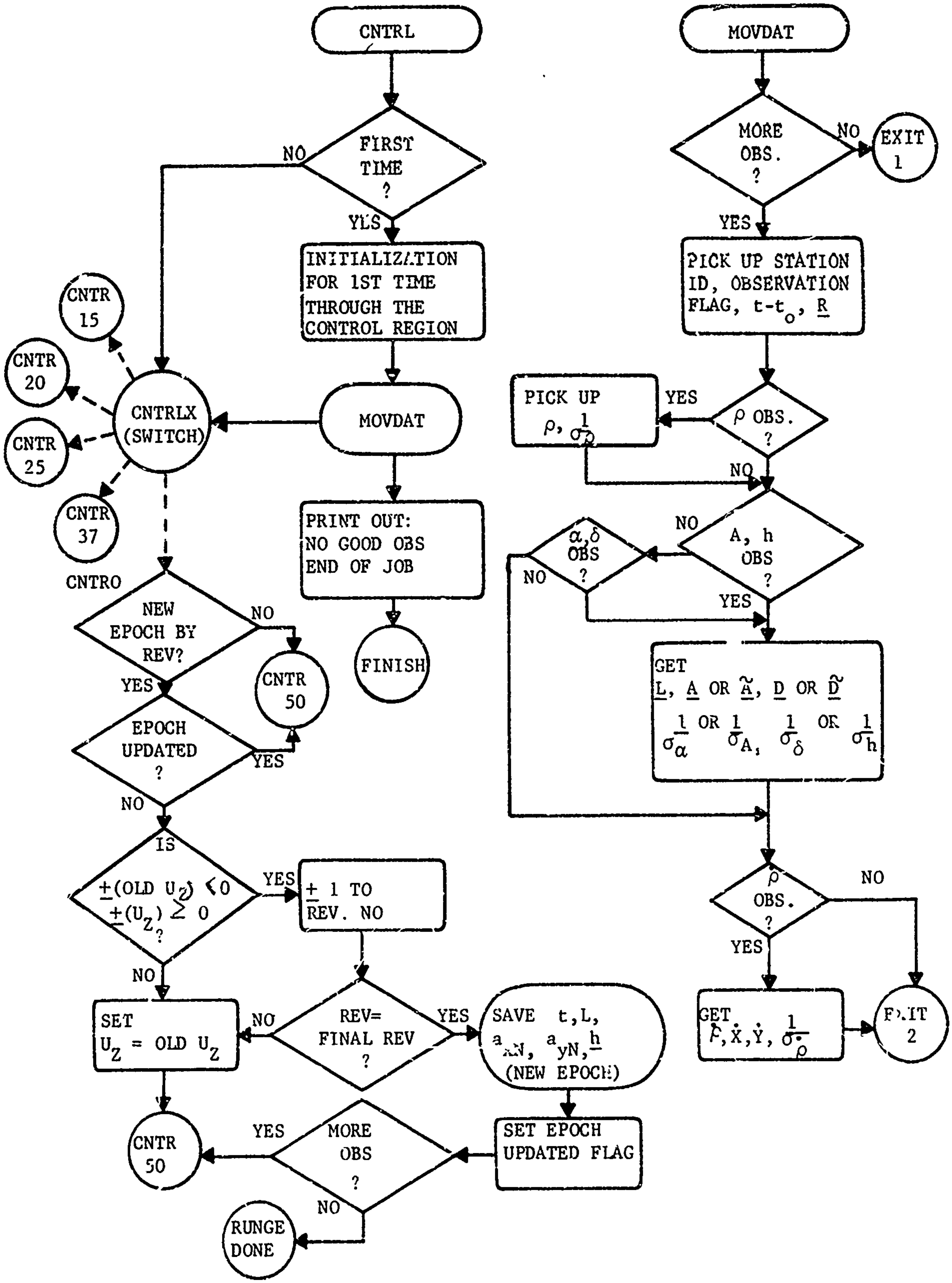


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

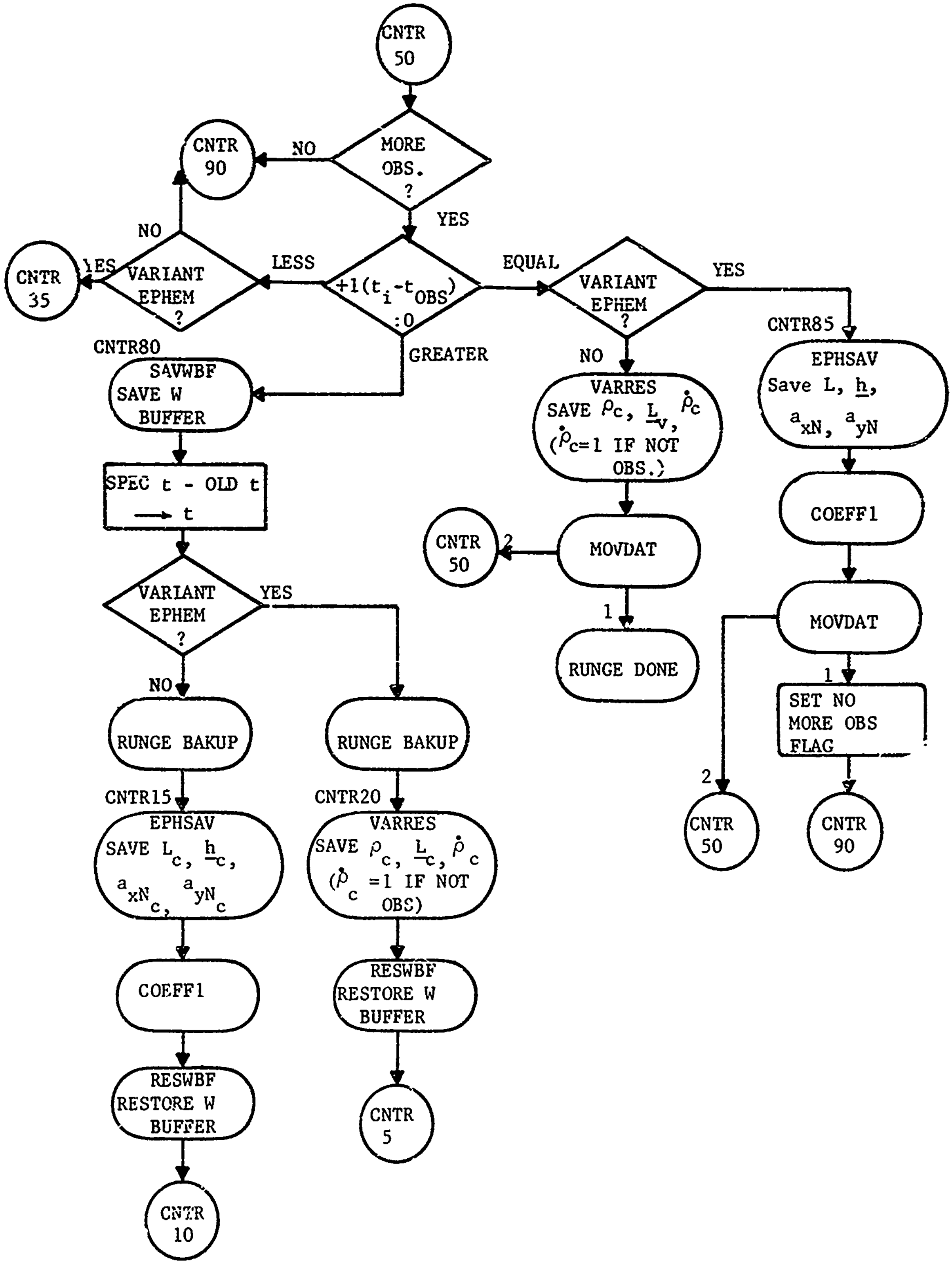


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

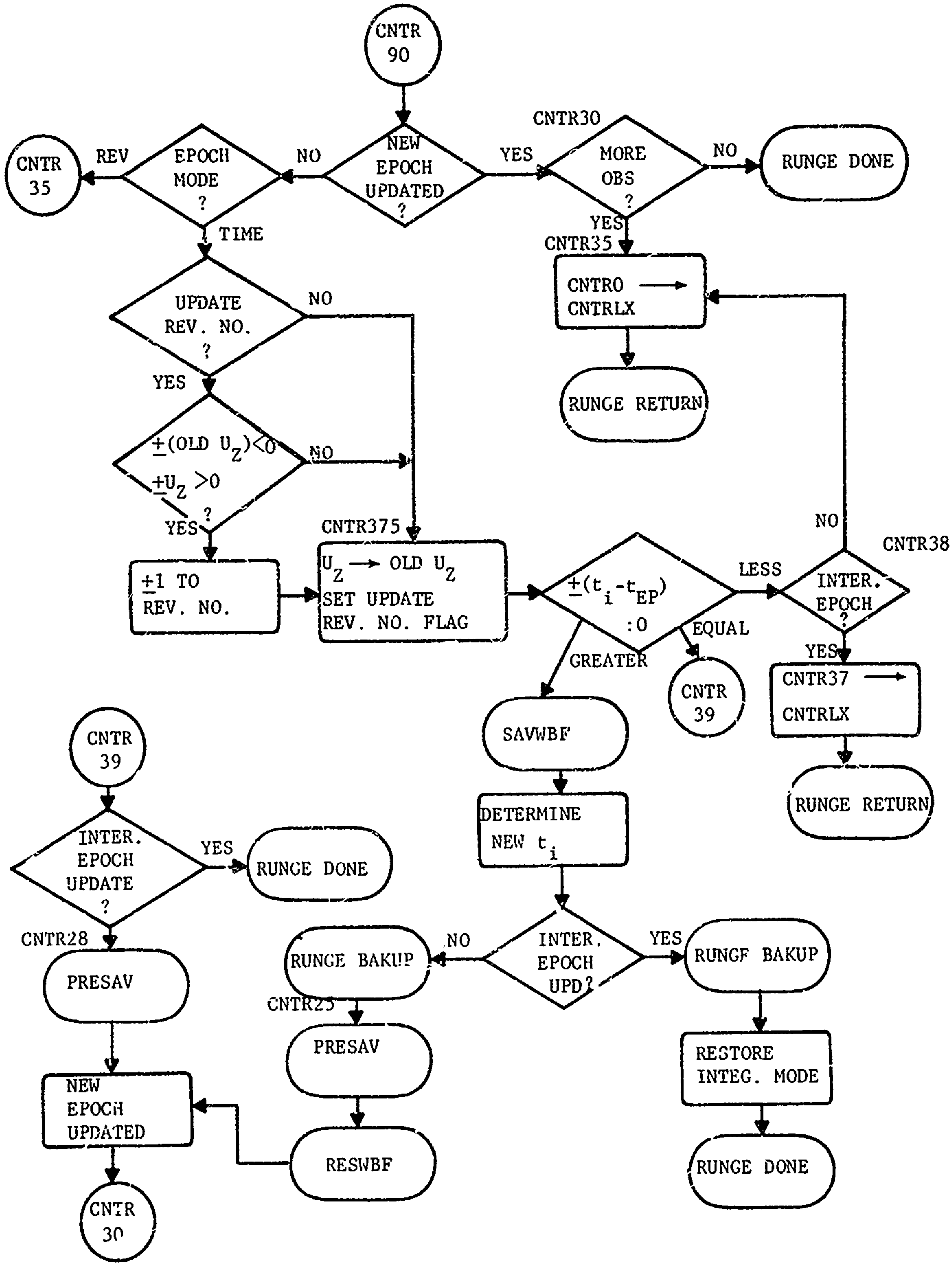


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

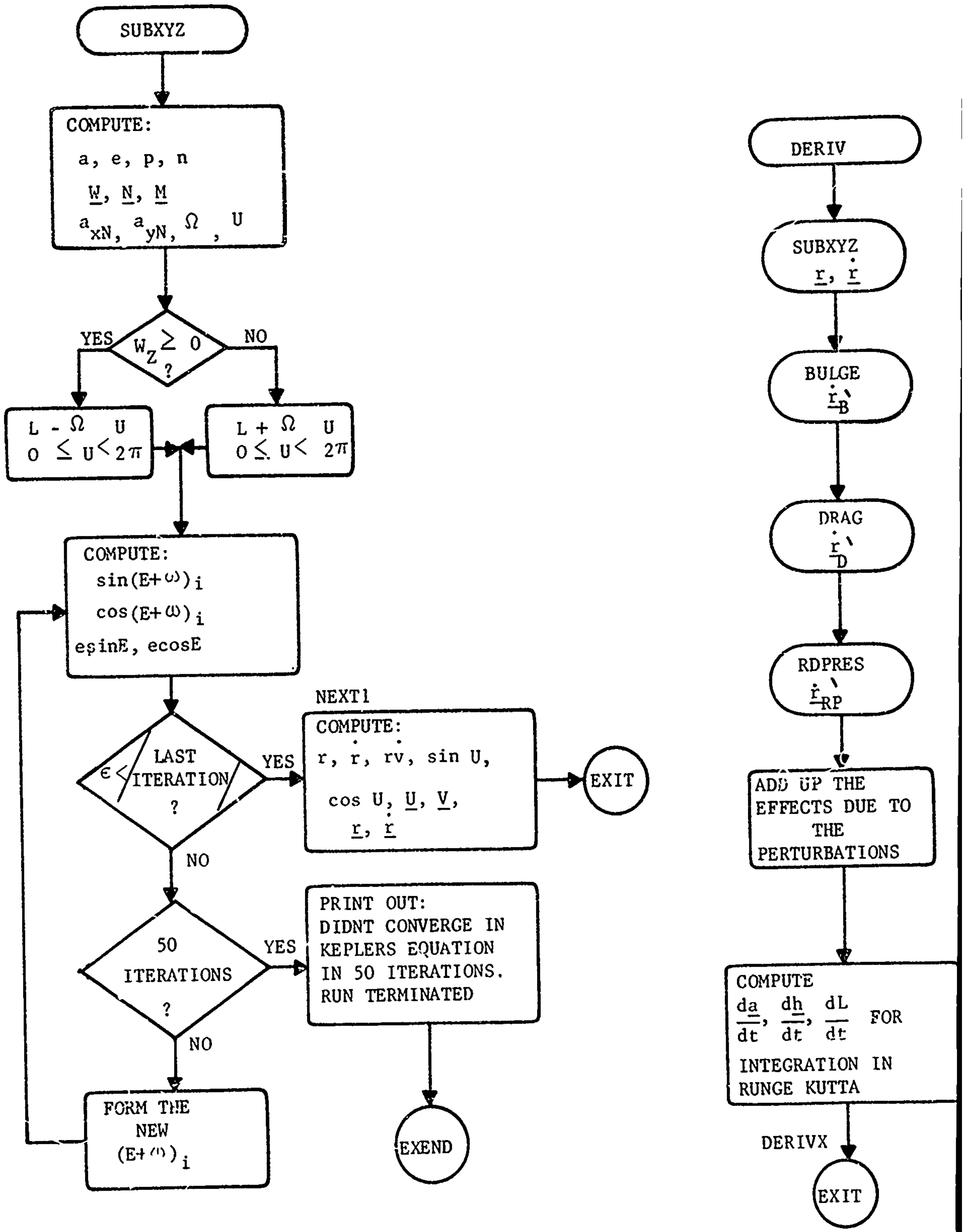


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

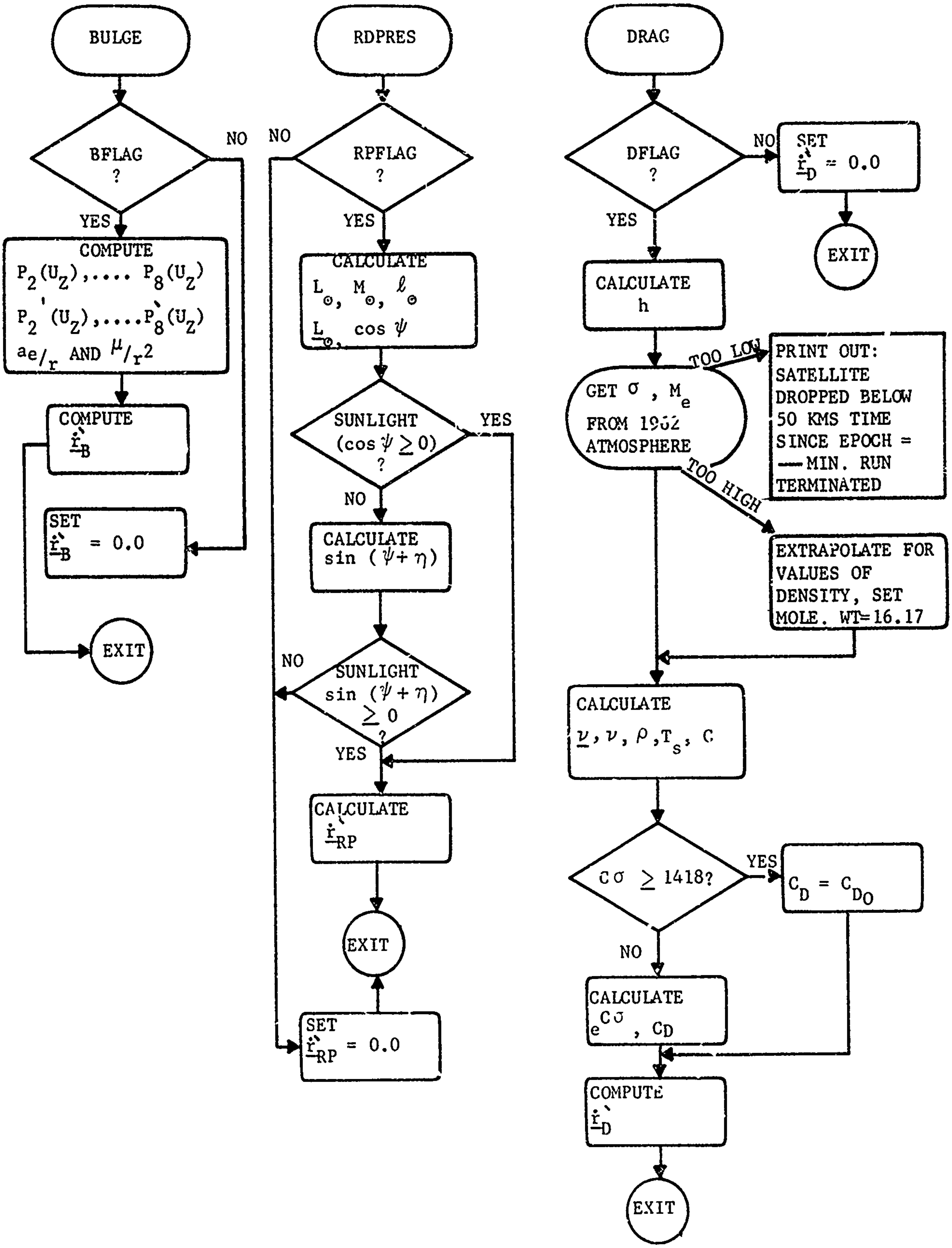


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

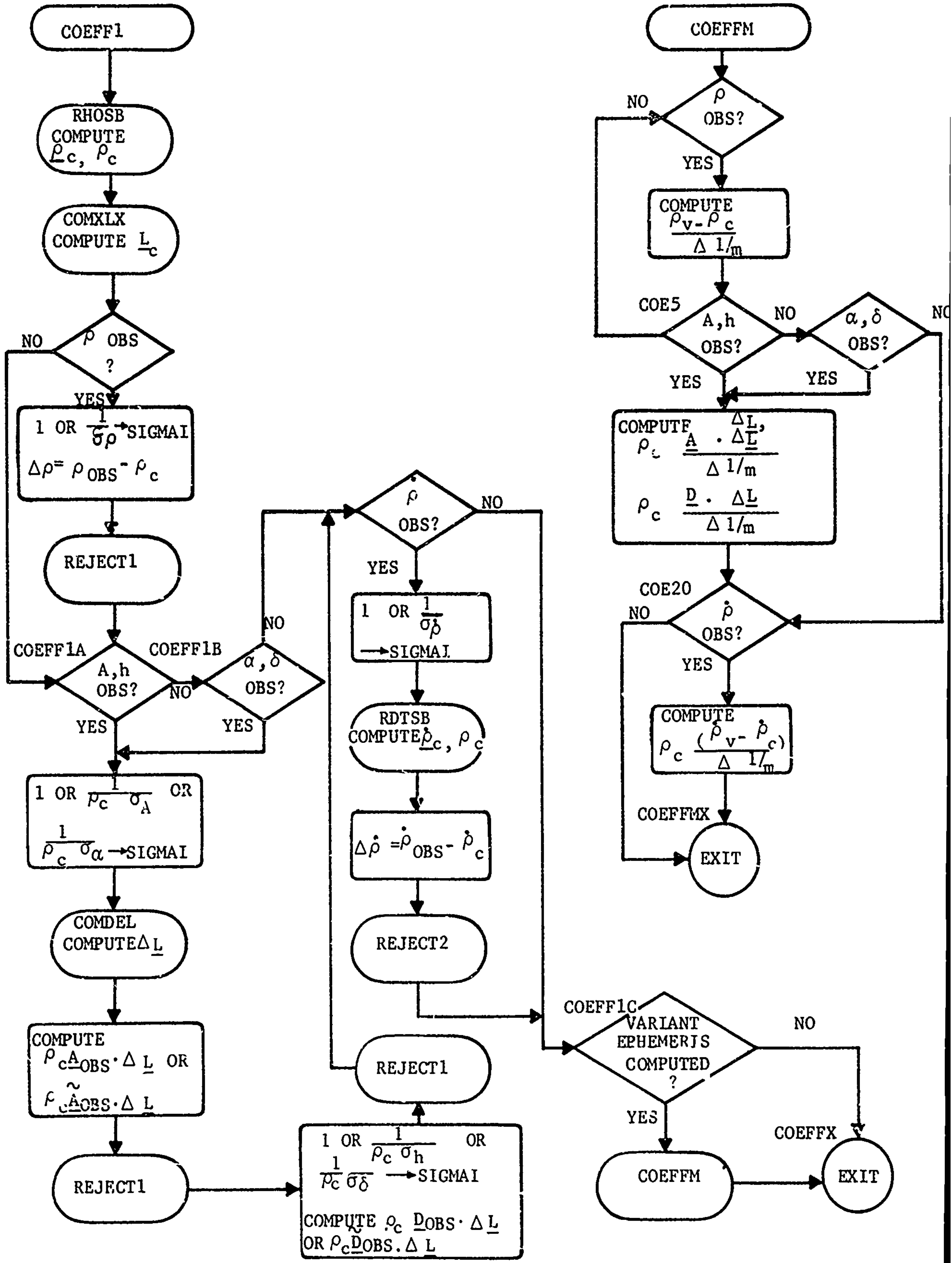


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

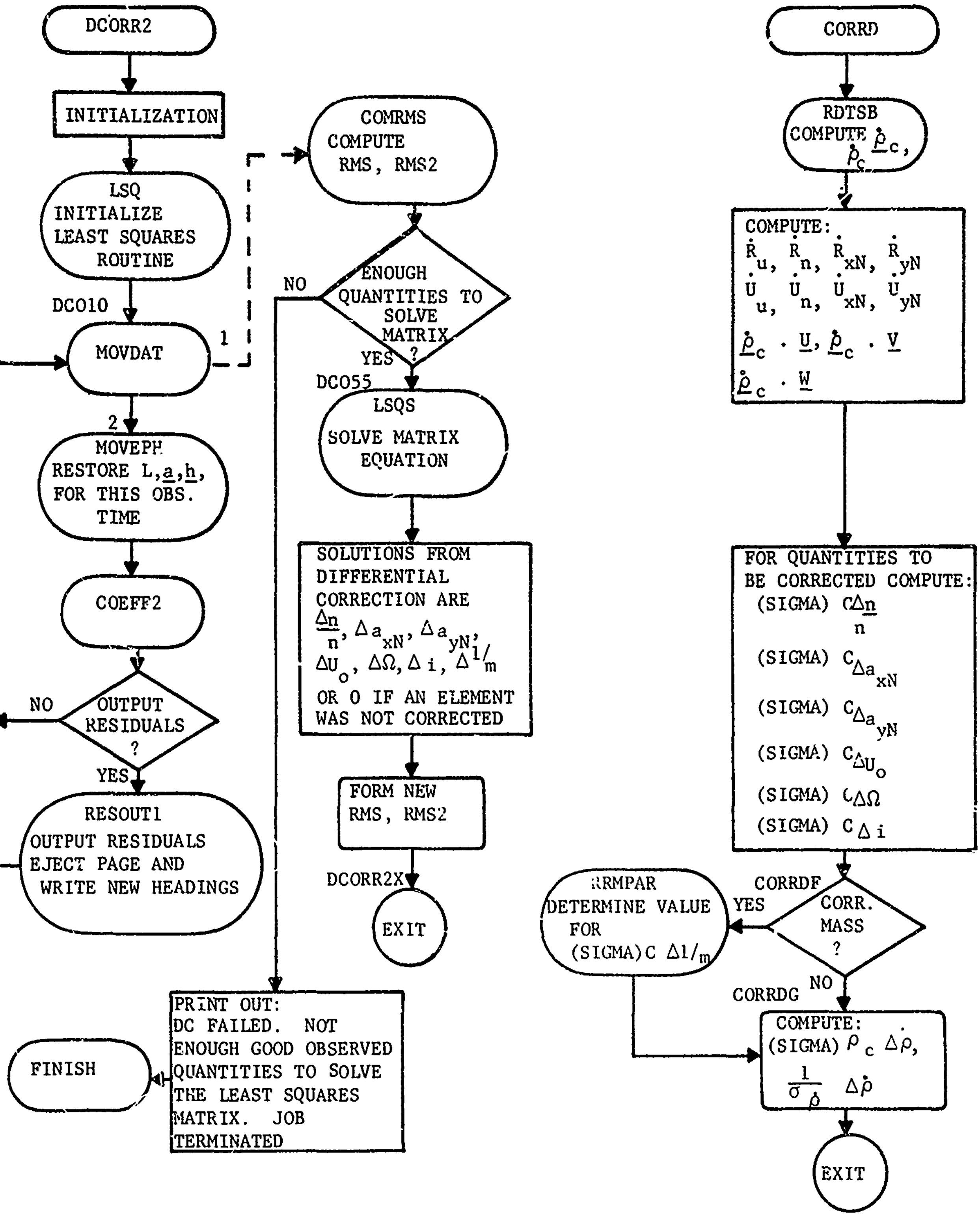


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

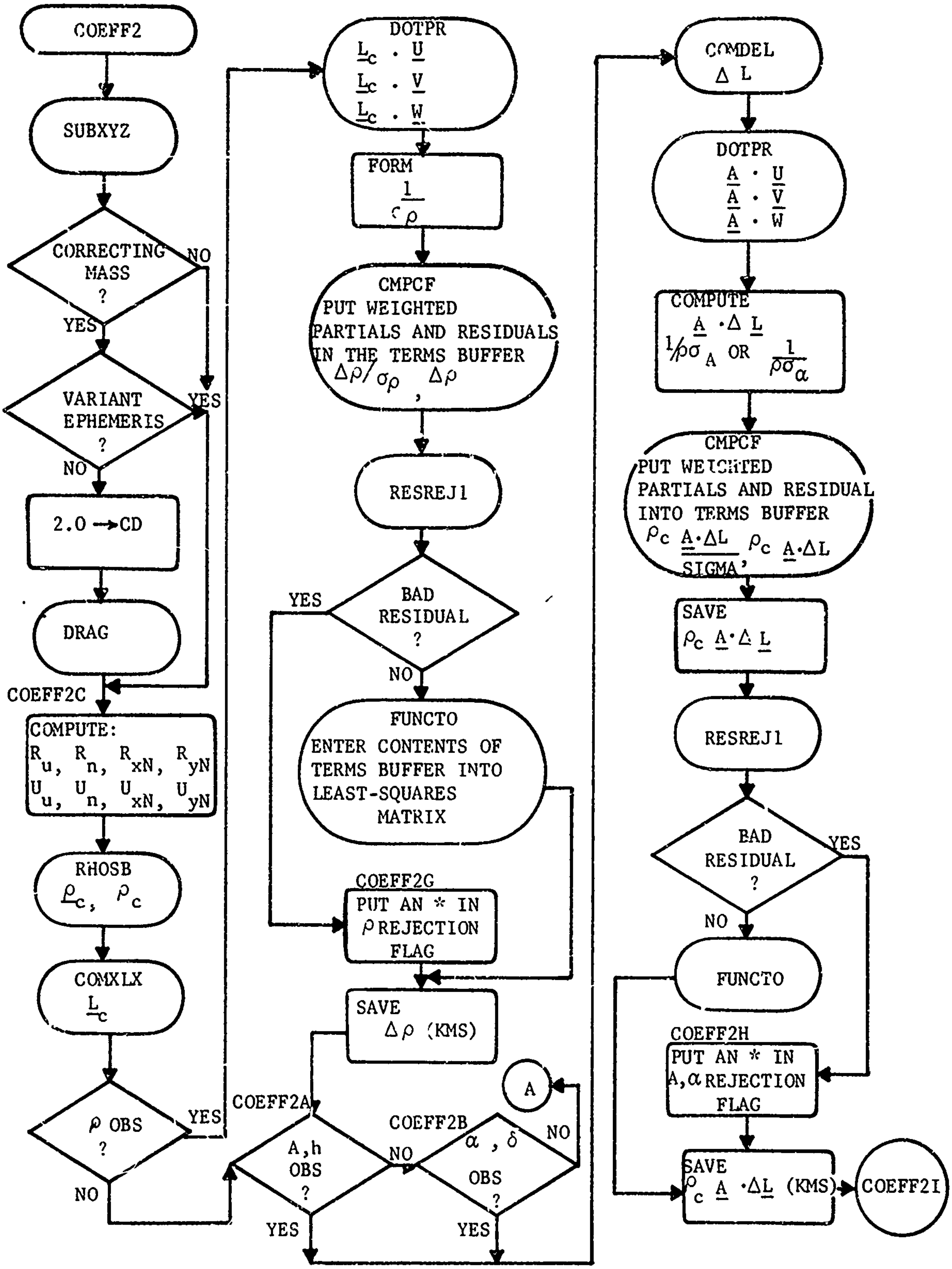


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

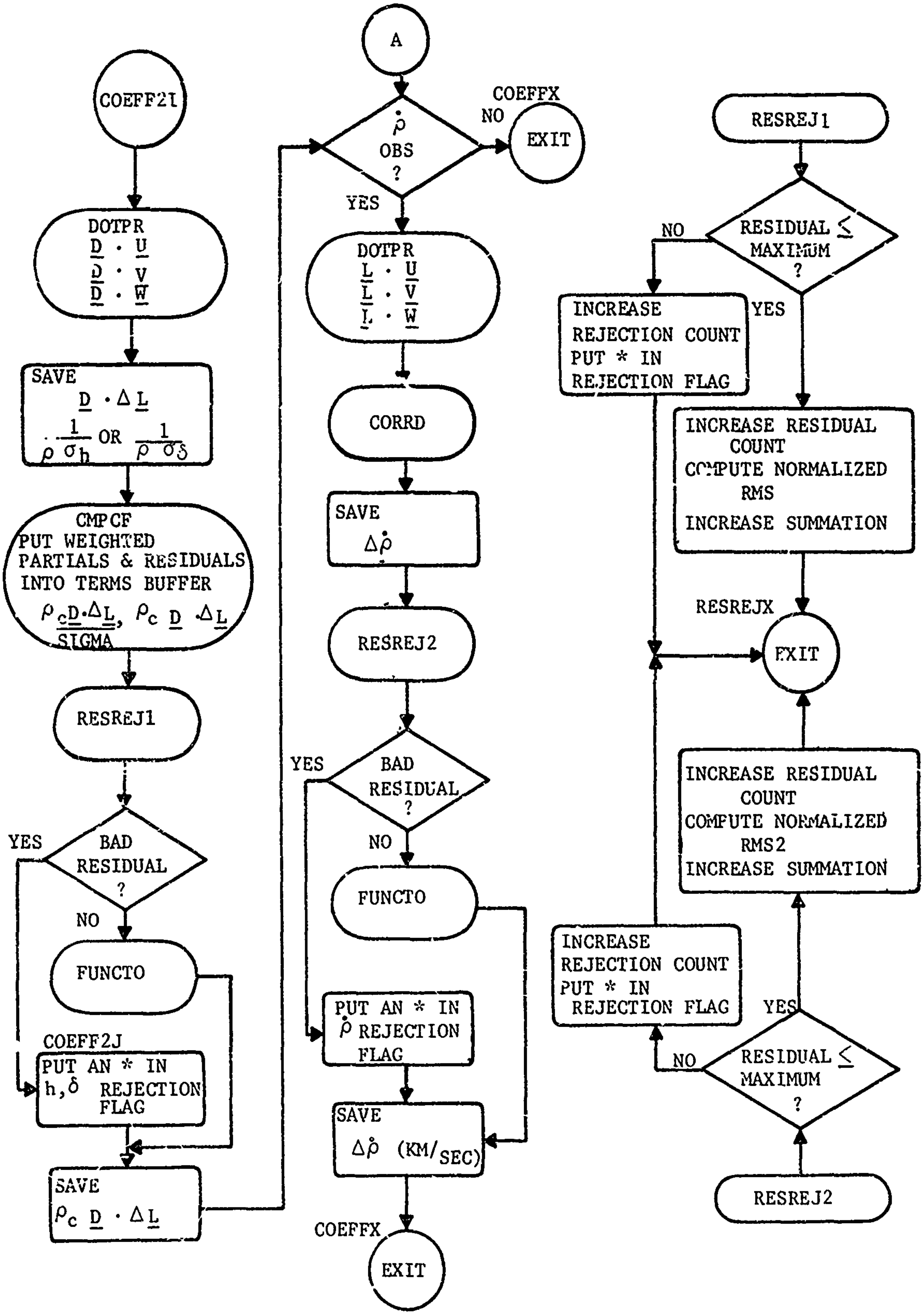


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

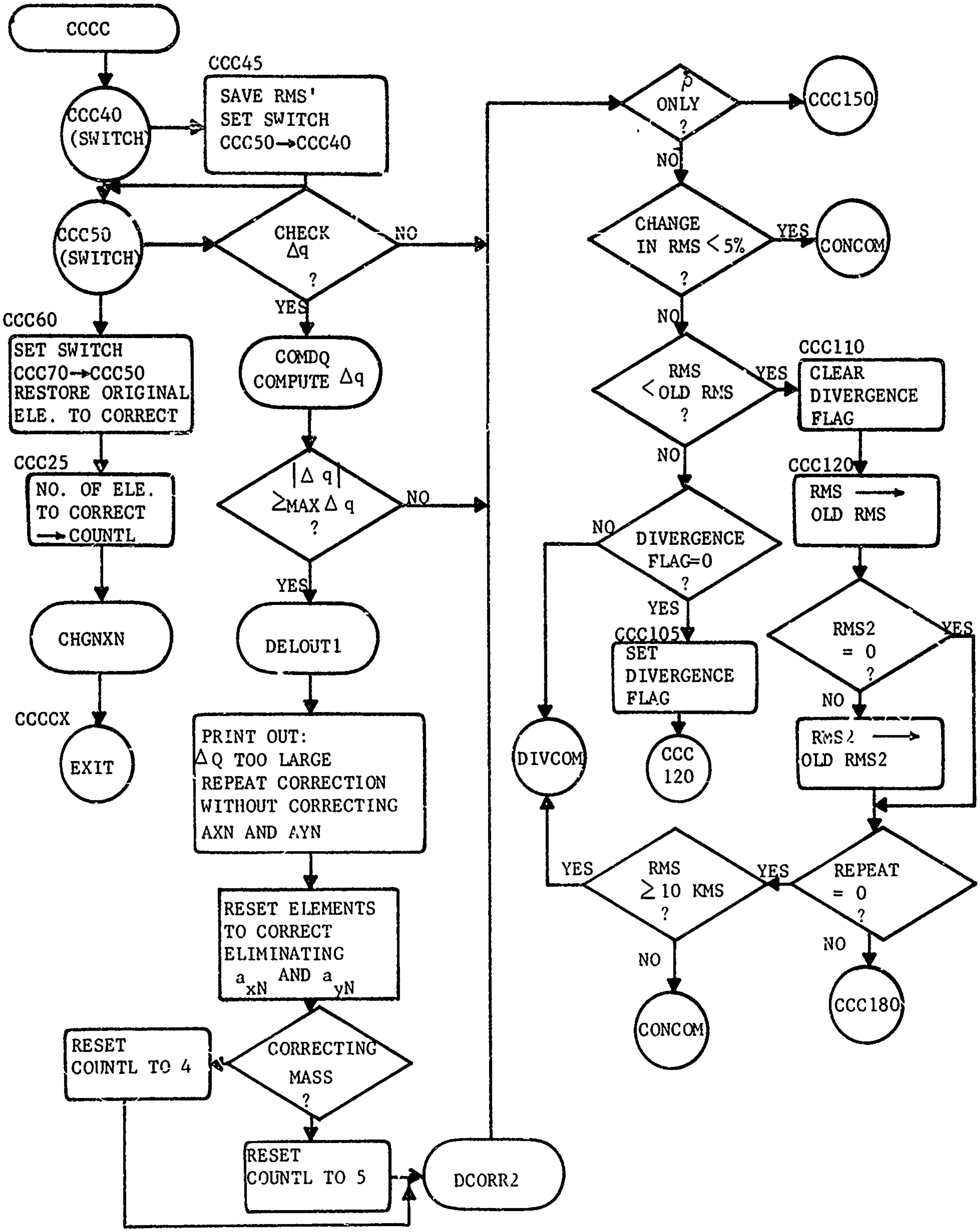


FIGURE 11. DETAILED FLOW DIAGRAMS (Continued)

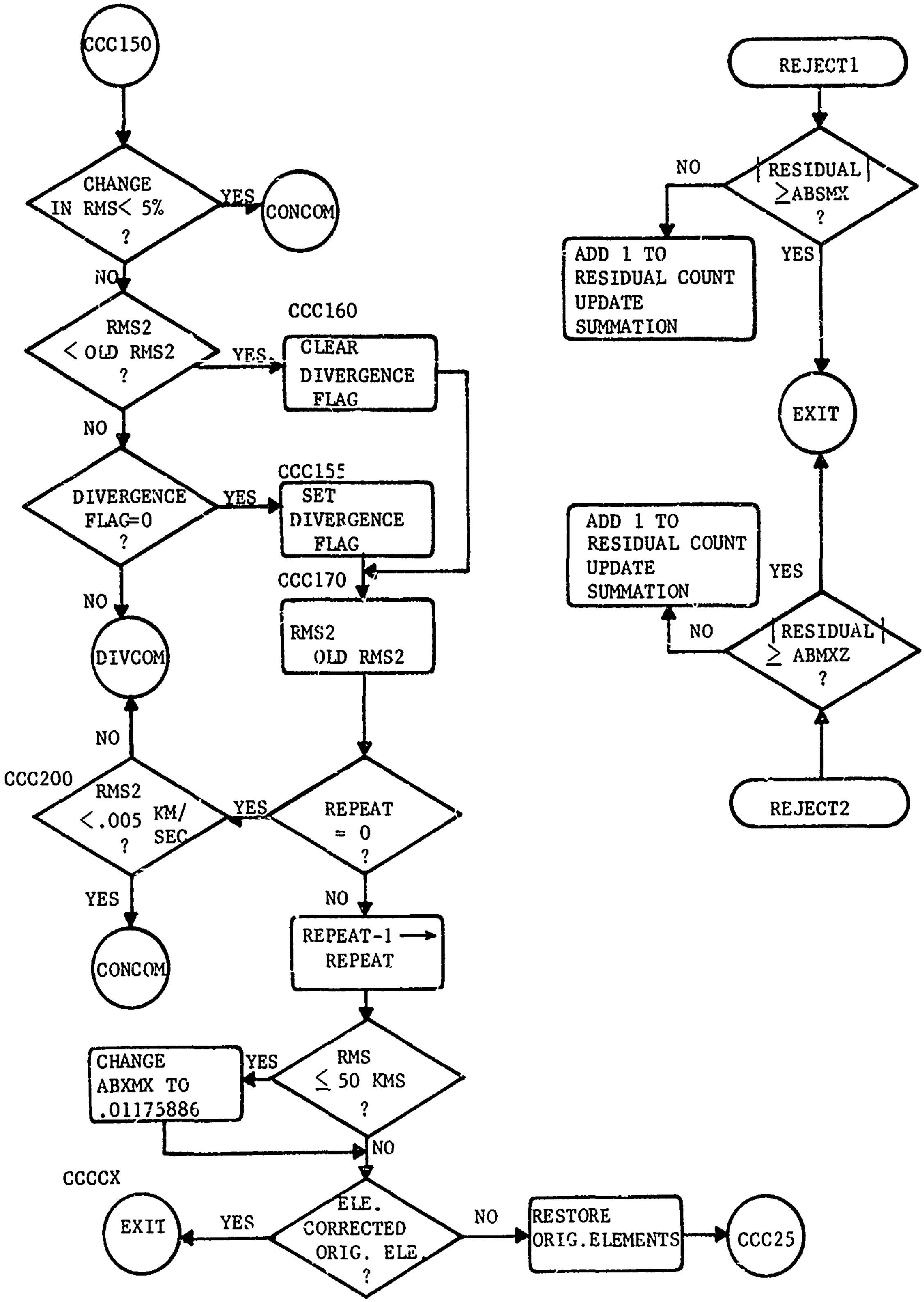


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

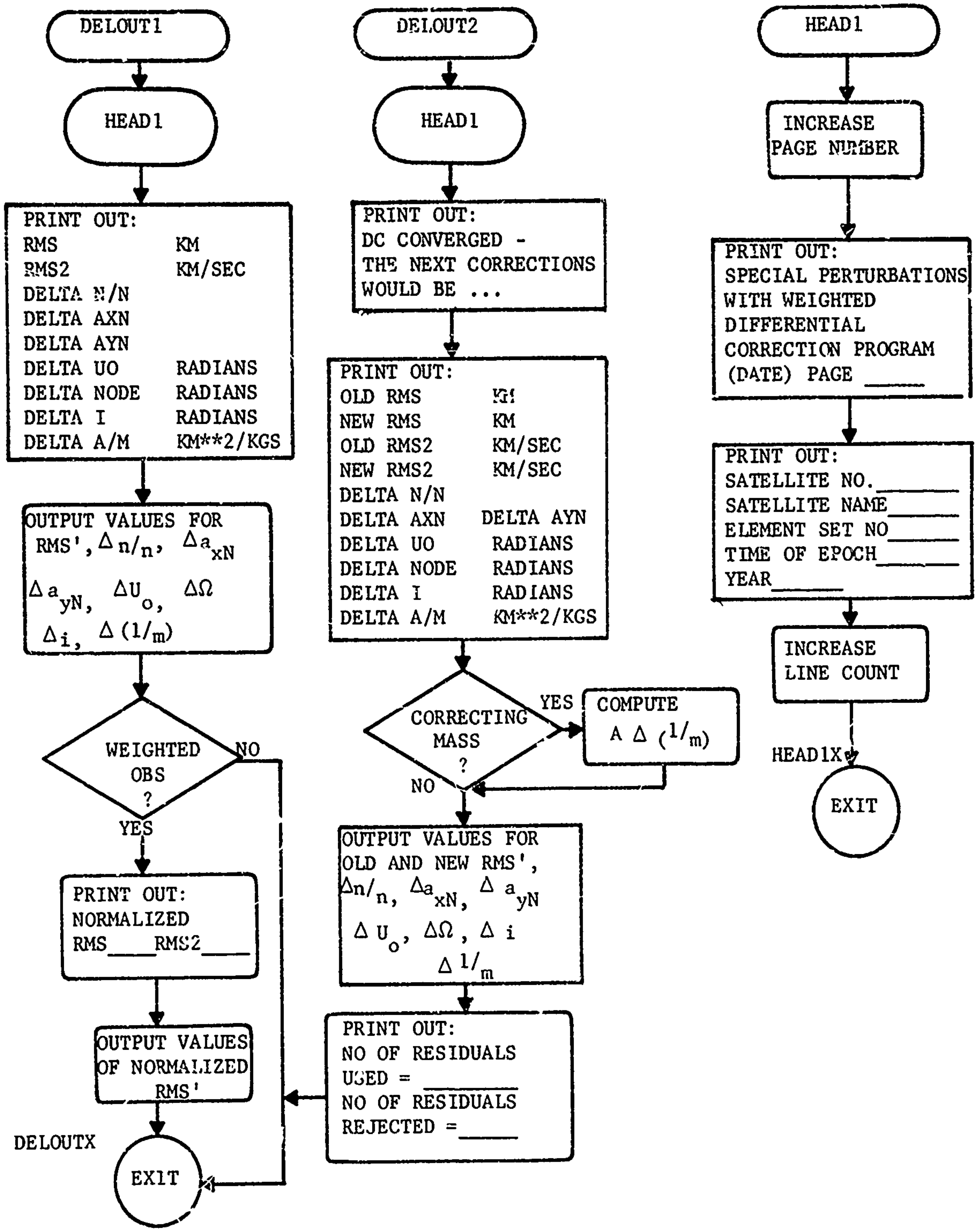


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

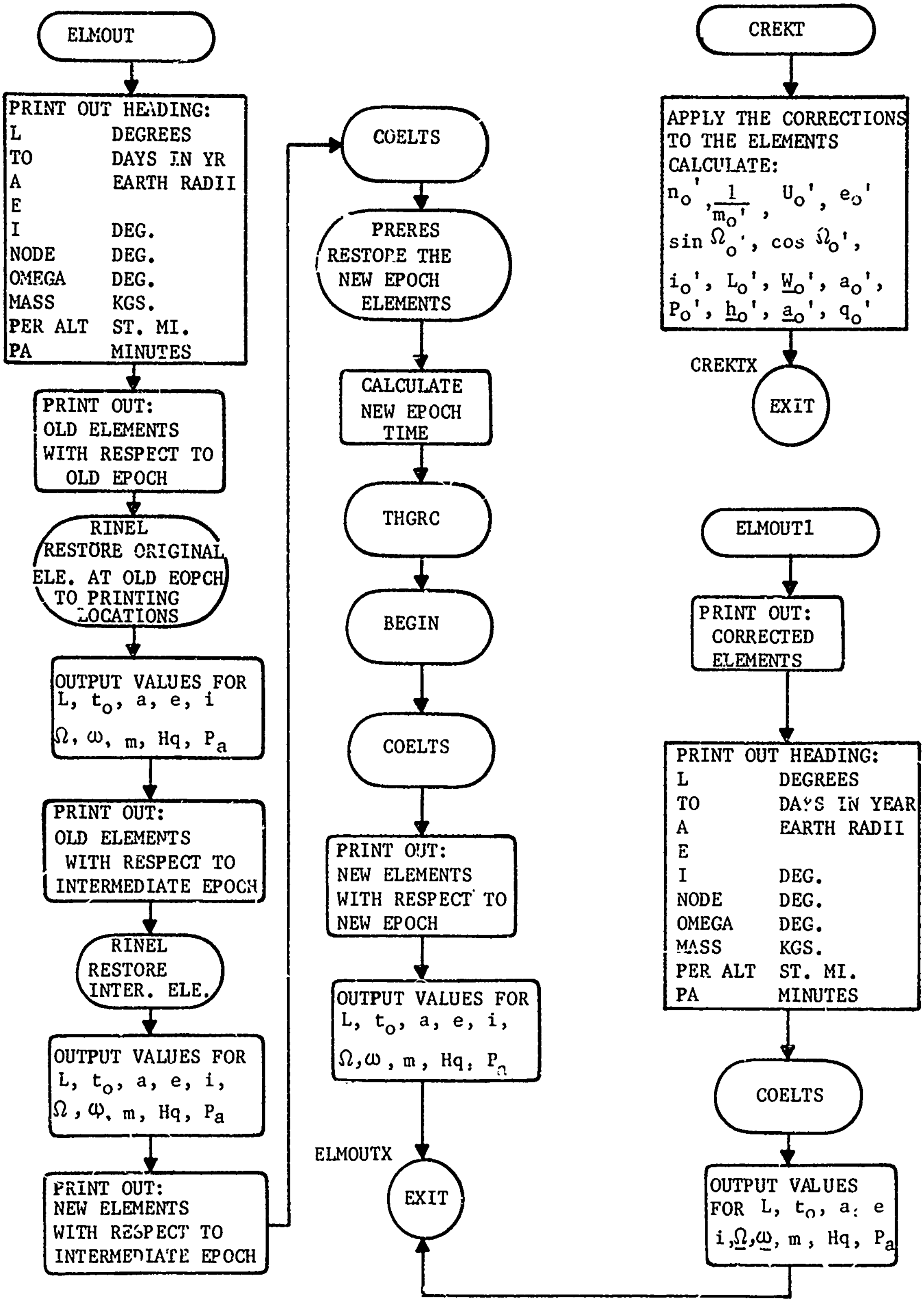


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

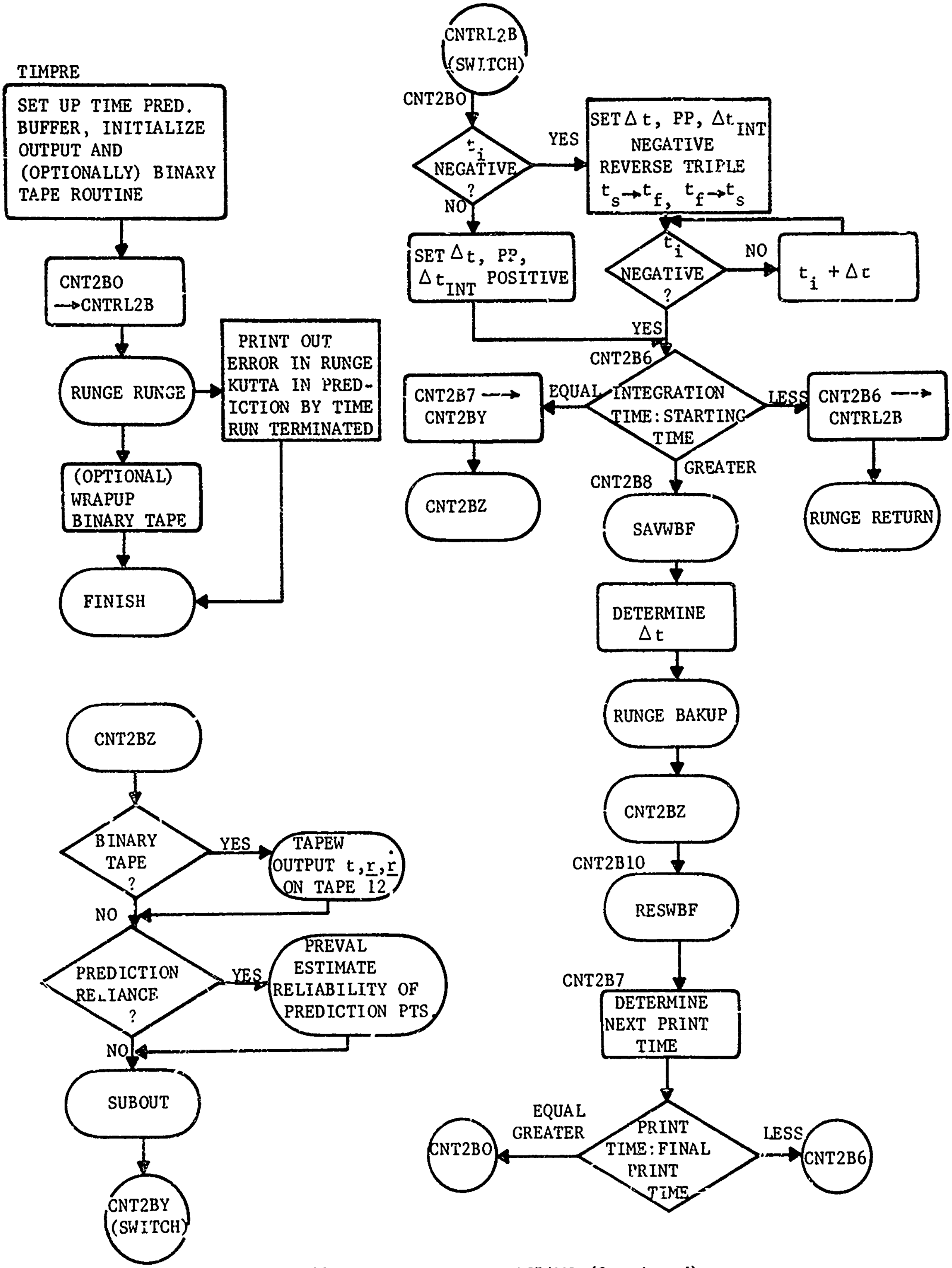


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

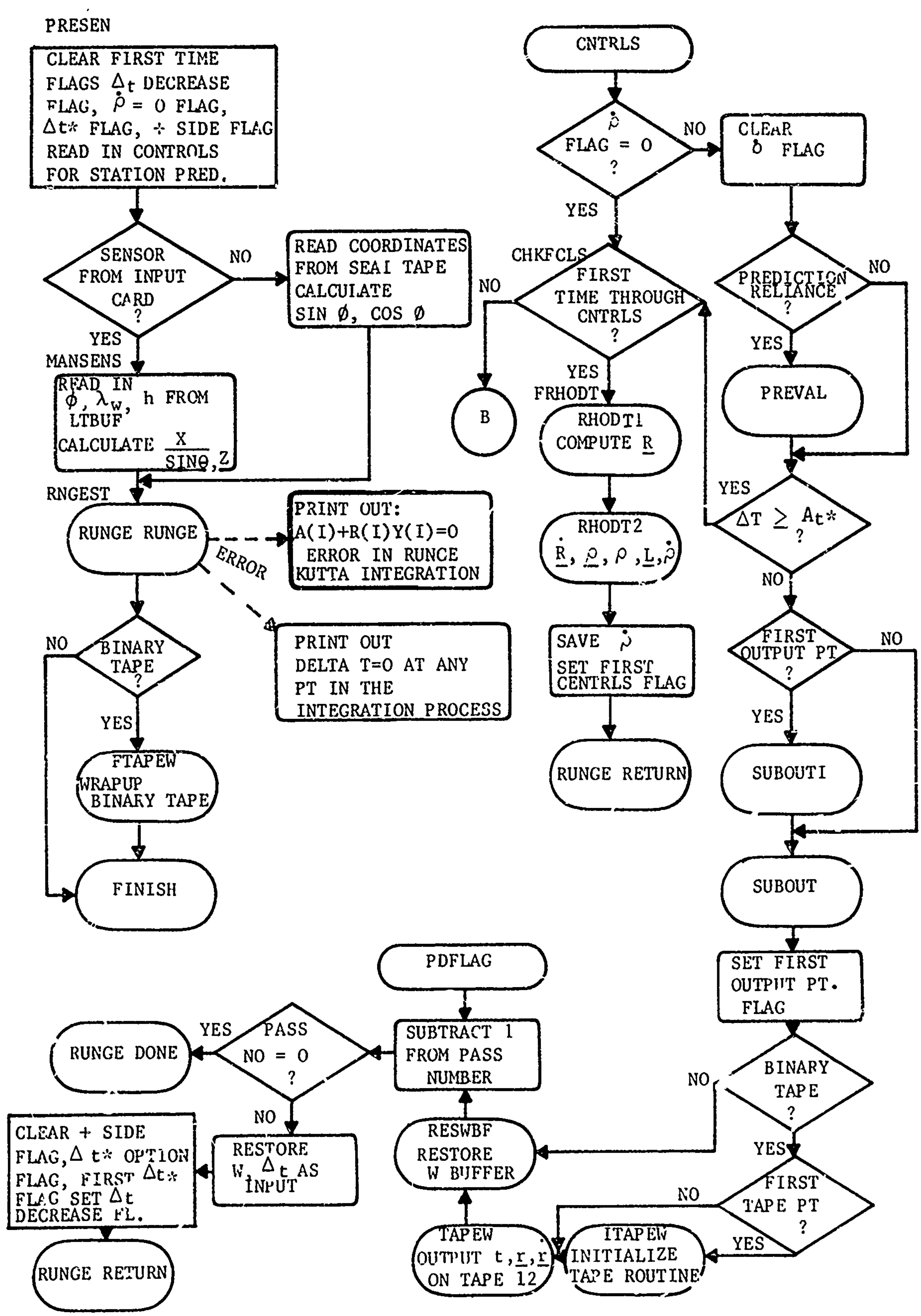


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

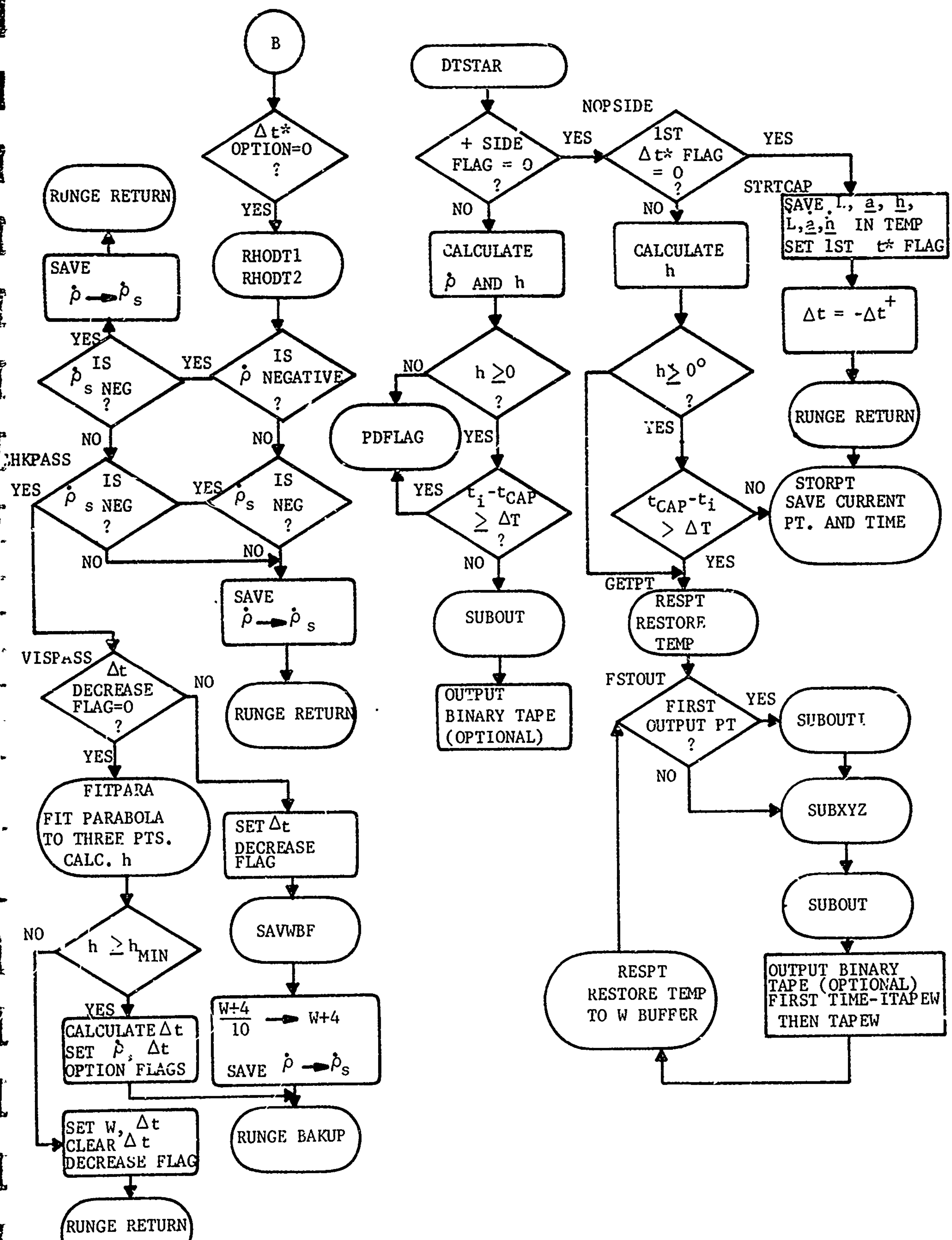


FIGURE 31. DETAILED FLOW DIAGRAMS (Continued)

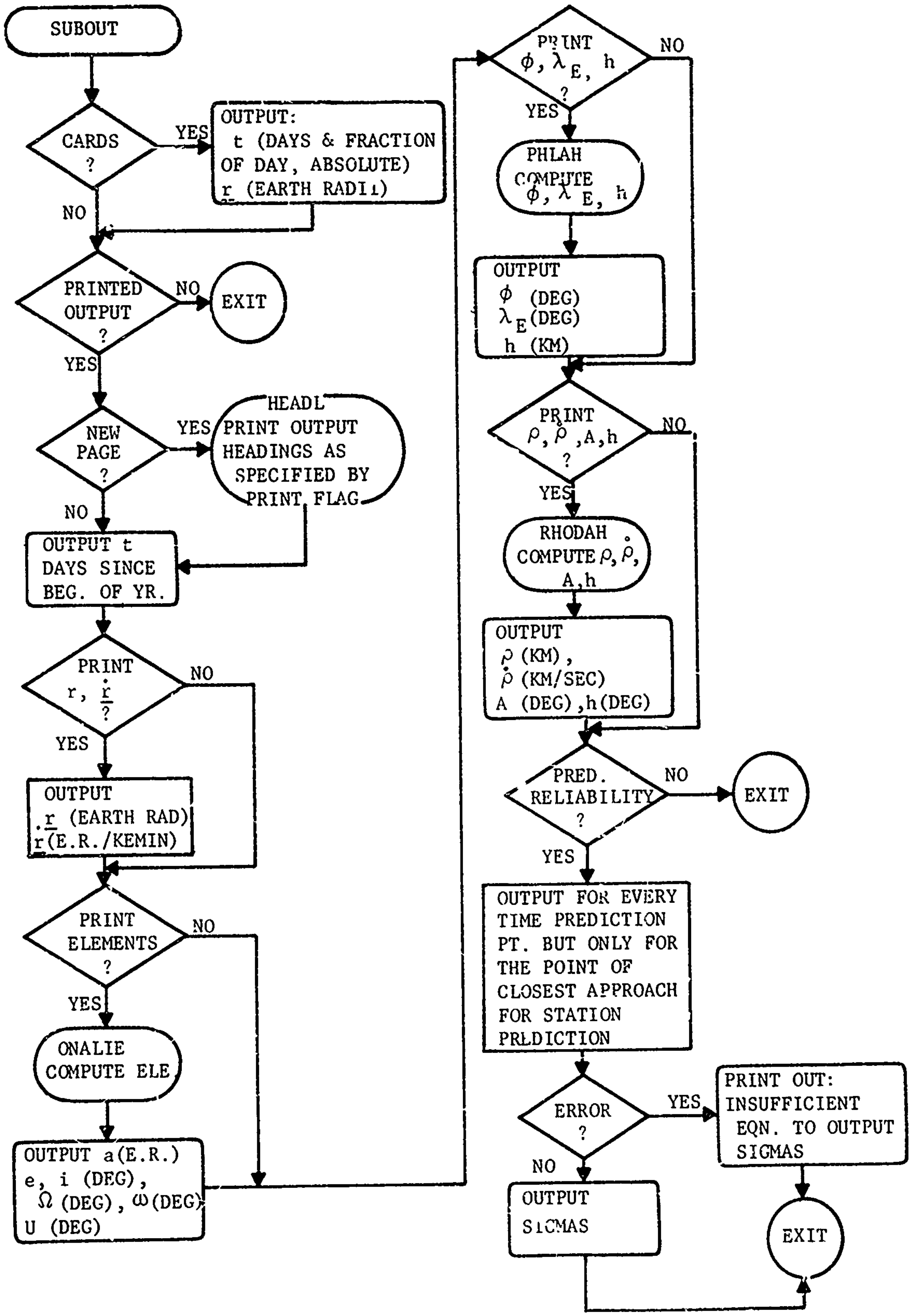


FIGURE 31 DETAILED FLOW DIAGRAMS (Continued)

APPENDIX III

PROGRAM GLOSSARY*

SYMBOL	DESCRIPTION
A	a
ACCCNT	Total residual count
AE	a_e
ALFLG	A, α residual flag
APRINT	Output location for a_o
AREA	Satellite area $(\pi \frac{d^2}{4})$
ASTK	*
AXGR	a'_x
AXO	a_{x_o}
AYGR	a'_y
AYO	a_{y_o}
AZGR	a'_z
AZIM	Azimuth
AZO	a_{z_o}
BFLAG	Bulge flag
BGOB	Reformatted <u>OBLOC</u> (16 word format)
BTFLAG	Binary tape flag
CAPD	D <u>(DERIV)</u>
CC	C <u>(DRAG)</u>
CCC40	Switch for <u>CCCC</u> first time

*Includes (1) SPWDC program symbols and (2) all symbols not appearing in the SPS B-2 Assign Deck. Underscored, capitalized notations refer to subroutines in the program.

APPENDIX C (Continued)

CCC50	Switch for CCCC n only correction on first pass
CDO	C_D = 0.92 (DRAG)
CD	C_{D_o} (DRAG)
CFLAG1	First time through flag (CNTRL)
CFLAG2	No more observations flag (CNTRL)
CFLAG3	Variant, normal ephemeris flag (CNTRL)
CFLAG4	New epoch by revolution or time flag (CNTRL)
CFLAG5	New epoch updated flag (CNTRL)
CNFLAG	Correct n flag
CNTRLX	Switch for D. C. Control Region (CNTRL)
CONTEST	Convergence test (5%) (CCCC)
COSEPS	cos ϵ = 0.9174469
COSPSI	cos ψ
COUNTLR	Same as COUNTL
COUNTL	Number of elements being corrected
COUNTR	Same as COUNTL
DCAPT	Δ T (Station Prediction)
DCFLAG1	Differential Correction flag
DCONT1	$\frac{V_{co}^3}{4\epsilon\sigma_s}\left(\frac{cm^3\ kemin^3\ {}^oK^4}{gm\ radii^3}\right) = 2.419012831\times10^{21}$
DCONT2	$\frac{-0.6378150\times10^{10}\ A}{2}\left(\frac{cm^3\ kg}{gm\ radii}\right)$
DC	D_c
DDGR	$\dot{D}$ (DERIV)
DELTAQ	Δq
DELTATO	Δt
DEN7	$\Delta(\frac{1}{m})$

DENOM	$1 + \sqrt{1 - e^2}$ (SUBXYZ)
DFLAG	Drag flag
DGR	D' (DERIV)
DIAM	Effective diameter of the satellite
DIVFL	Divergence flag
DLFLG	δ , h residuals flag
DLT1M	$\Delta (\frac{1}{m})$ (DCORR2)
DLVX	ΔV_x
DLVY	ΔV_y
DLVZ	ΔV_z
DQFLAG	Delta q check flag
DQN	Maximum Δq for Δq check
DRSDL	$\rho_c \underline{D} \cdot \Delta \underline{L}$ (kms)
DTASK	Δt^* (Station Prediction)
DTDCRS	Δt decrease flag (Station Prediction)
DT	Δt (prediction by time)
DTSROPT	Δt^* option flag (Station Prediction)
E2VGR	$-e^2 V'$
ECSIG	$e^{C\sigma}$
ELEV	h (RHODAH)
ELFLAG1	Error flag for prediction reliance
ELFLAG	Prediction Reliance Flag
EOCHK	Eccentricity check - variant ephemeris vs. analytical computation for mass

APPENDIX C (Continued)

EPHEM	Buffer for saving ephemeris L, a, h
EPRINT	Output location for e_o
EPS1	ϵ (RAD) for Kepler's Eqn = 10^{-8}
FOBHR	Output location for observation time - hr
FOBMIN	Output location for observation time - min
FOBSEC	Output location for observation time - sec
FRDAY	Integral days since beginning of year
FRFLAG	Punched card output flag (SUBOUT)
FRFRAC	Fractional day
FRSTFLG	First time flag (Station Prediction)
FRTIME	Days since beginning of year (SUBOUT)
FSTCTLS	First time flag (Station Prediction)
FSTDTSR	First Δ t* Pass flag (Station Prediction)
FSTPTW	First printed output flag
FSTSOUT	First printed output flag (Station Prediction)
GAMMA	Reflectivity of satellite
H2MH1	$H_2 - H_1$ (TBINT)
H2MH	$H_2 - H$ (TBINT)
HCON1	$3/2\ f^2 = 0.0168571736 \times 10^{-3}$ (CALH)
HMH1	$H - H_1$ (TBINT)

HMIN	h_{min} (Station Prediction)
HSUBQP	Output location for Hq_o (miles)
INELT2	Beginning storage location for original elements
INELT	Beginning storage locations for updated elements
INP20	Switch for Parameter Card Input (INPUT)
IPRINT	Output location for i_o (deg.)
JBUF	Kozai Values for J_2 through J_7
KEORTM	$Ke/\sqrt{\mu}$ = 0.07436574
KNTRL	Which elements to connect (m, i, Ω, U_o, a_{yN}, a_{xN}, n)
LINECT	Line Count
LPRINT	Output location for L_o (deg.)
LSQBUF	Least Squares Buffer
LSTT	Last t (PROOBS)
LTBUF	Buffer for Controls for prediction
MASSO	Original mass of satellite (Kgs)
MASSPR	Output location for mass
MASS	Program value of mass
MAX2	$\dot{\rho}$ rejection criteria

MAX	ρ and angle rejection criteria
ME	M_e TBINT
MFLAG1	Flag to compute mass correction analytically or by variant ephemeris
MSPRK	7905 (Prediction Reliance)
MU	μ
NLINE	Number of lines to be output
NOPASS	Pass number (Station Prediction)
NPRINT	Output location for Ω_o (deg.)
N	n (BULGE)
NU	ν
NUX	ν_x
NUY	ν_y
NUZ	ν_z
OBDAY	Day of Observation
OBFLG	Observation flag (What quantities were observed)
OBMO	Month of observation
OBYEAR	Year of observation
OFLAG4	Residual sets to output flag

APPENDIX C (Continued)

OFLAG5	Set up by OFLAG4 (Output residuals or not)
OLDRMS2	Old RMS for range rate
OLDRMS	Old rms for range and angles
OLDUZ	Old U_z
OMEGAP	Output location for ω_o (deg.)
OPRTESQ	$1 + \sqrt{1 - e^2}$
ORMS2	Old rms for range rate
ORMS	Old rms for range and angles
OUTAPE	Output tape number
PAGENO	Page number
PAPRINT	Output location for Pa_o (min.)
PBUF	P_2 through P_8 (BULGE)
PDAY	Output location for day
PDAY	Printing location for day (IHEAD1)
PFLAG	Prediction by Station pass or time flag
PHI	ϕ
PISUN	$\pi_{\odot} = 4.929316613$
PLSDE	Plus side flag (Station Prediction)
PNPRINT	P_N output location
PPBUF	P' Buffer (BULGE)
PPPFLAG	Prediction option input

PP	$\pm$ 1
PREDBF	Buffer to save new epoch time, L, $\underline{a}$, $\underline{h}$
PRTIME	Time of new epoch
PRTIM	Buffer for time prediction
PYEAR	Output location for year (<u>IHEAD1</u>)
QQ	Temporary location
RCNT2	Residual count for $\dot{\rho}$
RCNT	Residual count for ρ and angles
RDOTOR	$\dot{r}/r$
RDTFLG	$\dot{\rho}$ = 0 flag (Station Prediction)
REJCNT	Number of residuals rejected
REJFLG	Rejection flag if any observation quantity is rejected
REV	Revolution number
RGFLG	Range residual flag
RGSDL	$\Delta\rho$ (km)
RHODT3	Storage location for <u>RHODT</u>
RHOO	ρ_o $({}^{gm}/{}_{cm^3})$ = 0.001225
RHO	ρ (<u>DRAG</u>)
RMS2	RMS for range rate

APPENDIX C (Continued)

RMSCHK	50 km
RMS	Root mean square for ρ, and angles
ROVA	r/a
RPCON1	$2e_{\odot} = 0.03345100$
RPCON2	$5/4\, e_{\odot}^2 = 0.0003496779$
RPCON3	$F_{\odot} = \gamma\, P_{\odot}\, A \frac{\text{Kg radii}}{\text{kemin}^2}$
RPFLAG	Radiation pressure flag
RPT	Number of passes through D.C.
RRFLG	Range rate residual flag
RRSDL	$\Delta\dot{\rho}$ (km/sec)
RSFLAG	Flag for CNTRL for intermediate epoch
RSFLG2	First time through CNTRL for intermediate epoch
RSFLG3	Storage location WFLAG (RESRET1)
SATEL	Satellite number
SATL	Output location for Satellite Number
SAVBUF	Buffer for saving W buffer before going to Runge-Kutta back-up routine
SAVEO	Storage location for cell 0
SAVE3	Storage location for cell 3
SAVEM	$\underline{r}_{OBS} \cdot \underline{W} \sqrt{\underline{r}_{OBS} \cdot \underline{r}_{OBS}}$
SENSNO	Sensor number (Station Prediction)

APPENDIX C (Continued)

SETRD2	Address after last cell in processed observation buffer
SETRD	Starting address of processed observation buffer
SIGMA1	$\frac{1}{\sigma_\rho}$
SIGMA2	$\frac{1}{\sigma_{\dot\rho}}$
SIGMA3	$\frac{1}{\sigma_\alpha}$ or $\frac{1}{\sigma_A}$
SIGMA4	$\frac{1}{\sigma_\delta}$ or $\frac{1}{\sigma_h}$
SIGMAI	$\frac{1}{\sigma_\rho}$ for ρ_{OBS}, $\frac{1}{\rho_c \sigma}$ for angles observed
SIGMA	σ (DRAG) (TBINT)
SINBOTH	$\sin(\psi + \epsilon)$
SINEPS	$\sin \epsilon = 0.3978584$
SKNTRL	Storage location for KNTRL
SMLGR	ℓ' (DERIV)
SOBFLG	Storage location for OBFLG
SOFLAG5	Storage location for OFLAG5
SV56	Storage location for index registers 5 and 6
SVDLTTO	Storage location for DELTATO
SVPRTIM	Storage location for PRTIME
SVRDTBD	Storage location for RHODT
SVRHODT	Save $\dot{\rho}$ (Station Prediction)
SWFLAG	Set up by WFLAG

APPENDIX C (Continued)

TCAP	T (Station Prediction)
TEMP1	Temporary
TEMP2	Temporary
TEMP3	Temporary
TEMP	Storage location for PRTIME
TERMS	Least Squares Buffer
TF	t_f (final output time - time pred)
THETA	θ
TLFLAG	Point of closest approach flag (Station Prediction)
TMPEND	Temporary Ending address (Station Prediction)
TMPLOC1	Temporary
TMPLOC2	Temporary
TOPR	Same as TO
TOYPR	Output location for epoch year
T	t (minutes since epoch)
TSI	Beginning time point (Time Prediction)
TS	Ts (^{o}K)
VFLAG1	Variant Ephemeris Flag

WFLAG	Weight Flag	
W	Fixed or Variable integration mode (Runge-Kutta)	
WSUM2	Normalized RMS for range rate	
WSUM	Normalized RMS	
X1OMO	$\frac{1}{m_o}$	
XBDGR	$\dot{x}_B$	(BULGE)
XDGR	$\dot{x}_B$	(DERIV)
XDTGR	$\dot{x}_D$	(DRAG)
XLAMD	λ_E	
XLSNSBX	$L_{x\odot}$	(RDPRES)
XLSNSBY	$L_{y\odot}$	(RDPRES)
XLSNSBZ	$L_{z\odot}$	(RDPRES)
XLSUN	$L_{\odot}$ (rad)	(RDPRES)
XLSUNT	$\ell_{\odot}$ (rad)	(RDPRES)
XMPER	$^{Kr}/_{radii}$	
XMSUN	$M_{\odot}$ (rad)	(RDPRES)
XNSUN1	$n_{\odot}$ $(^{deg}/_{day}) = 0.9856473354$	(THGRC)
XNSUN	$n_{\odot}$ $(^{rad}/_{min}) = 0.01194638282 \times 10^{-3}$	(RDPRES)
XRDGR	$\dot{x}_{RP}$	(RDPRES)

APPENDIX C (Continued)

YBDGR	$\dot{y}_B^{\grave{}}$	(<u>BULGE</u>)
YDGR	$\dot{y}^{\grave{}}$	(<u>DERIV</u>)
YDTGR	$\dot{y}_D^{\grave{}}$	(<u>DRAG</u>)
YRDGR	$\dot{y}_{RP}^{\grave{}}$	(<u>RDPRES</u>)
ZBDGR	$\dot{z}_B^{\grave{}}$	(<u>BULGE</u>)
ZDGR	$\dot{z}^{\grave{}}$	(<u>DERIV</u>)
ZDTGR	$\dot{z}_D^{\grave{}}$	(<u>DRAG</u>)
ZRDGR	$\dot{z}_{RP}^{\grave{}}$	(<u>RDPRES</u>)

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